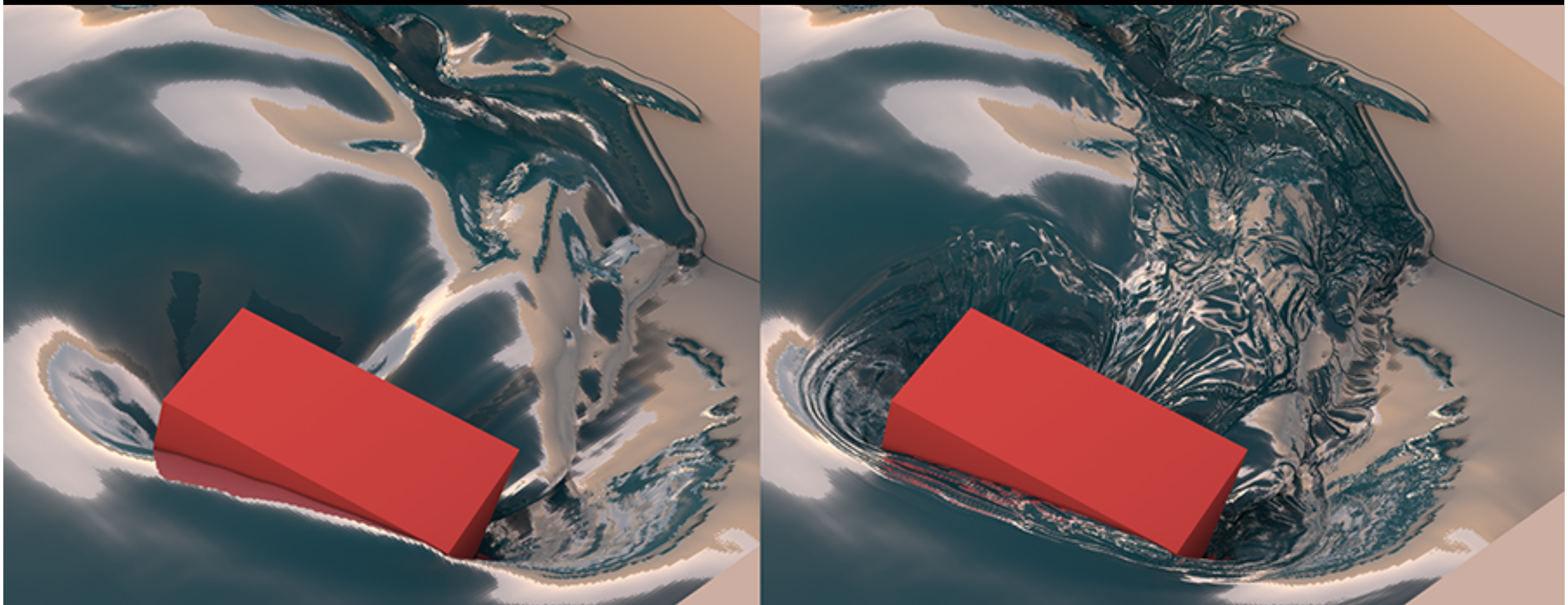
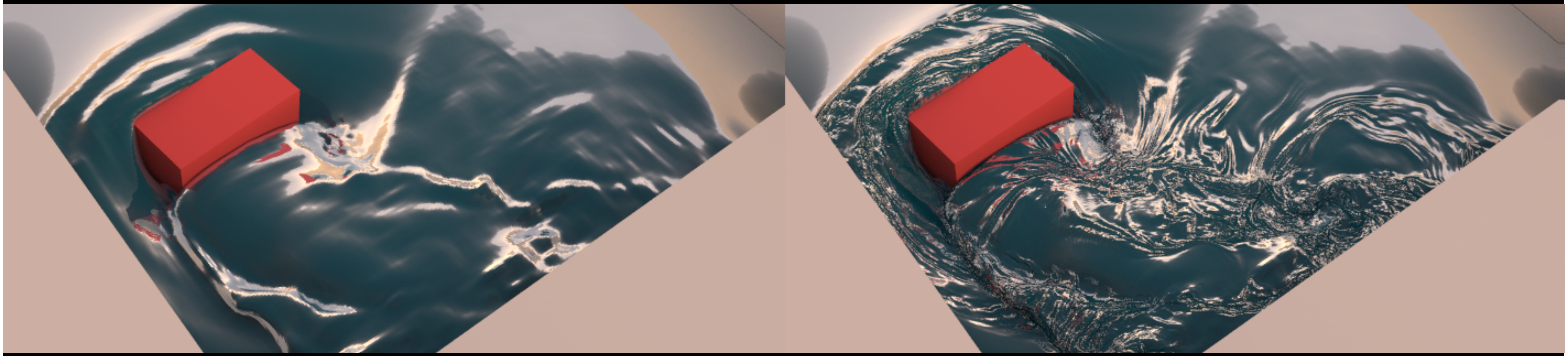


Part 3: Free Surface Turbulence

Theodore Kim

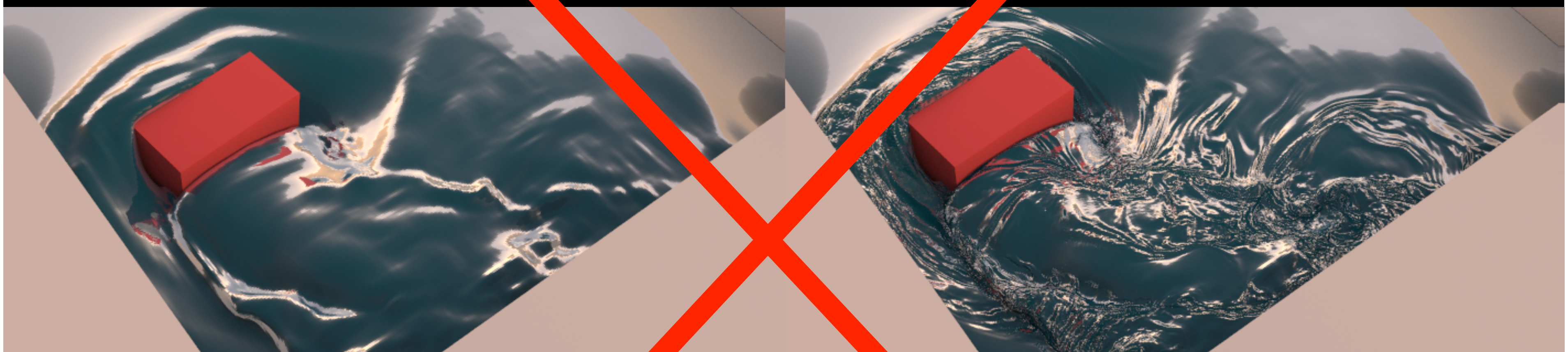
University of California, Santa Barbara



Closest Point Turbulence for Liquid Surfaces

Theodore Kim, Nils Thuerey, Jerry Tessendorf

ACM Transactions on Graphics, 2013

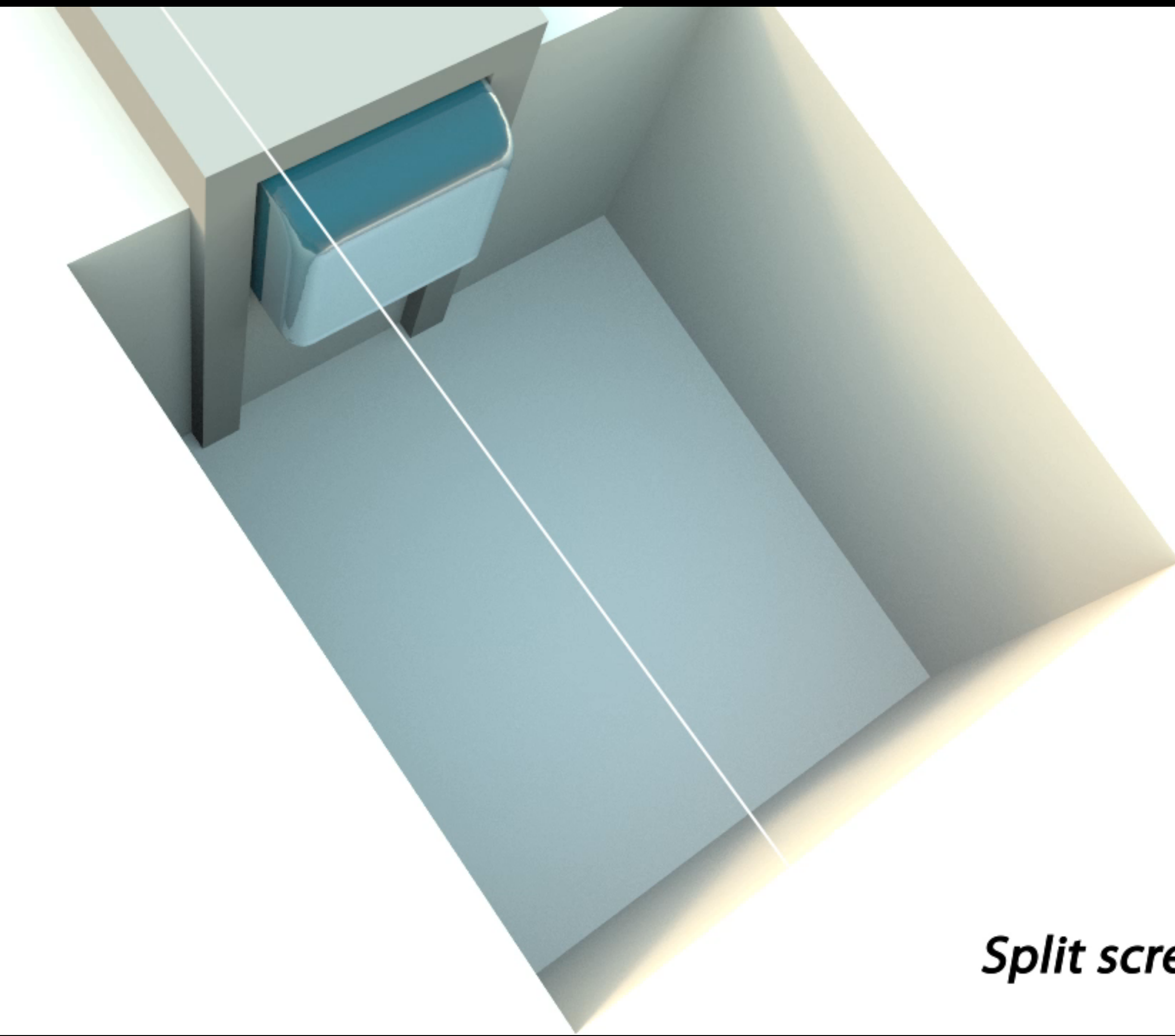


Closest Point Turbulence for Liquid Surfaces

Theodore Kim, Nils Thuerey, Jerry Tessendorf

ACM Transactions on Graphics, 2013

The Up-Res Workflow



Split screen comparison

Overview

- Related Works
- Wave Models
- The iWave Model
- Source Code

Overview

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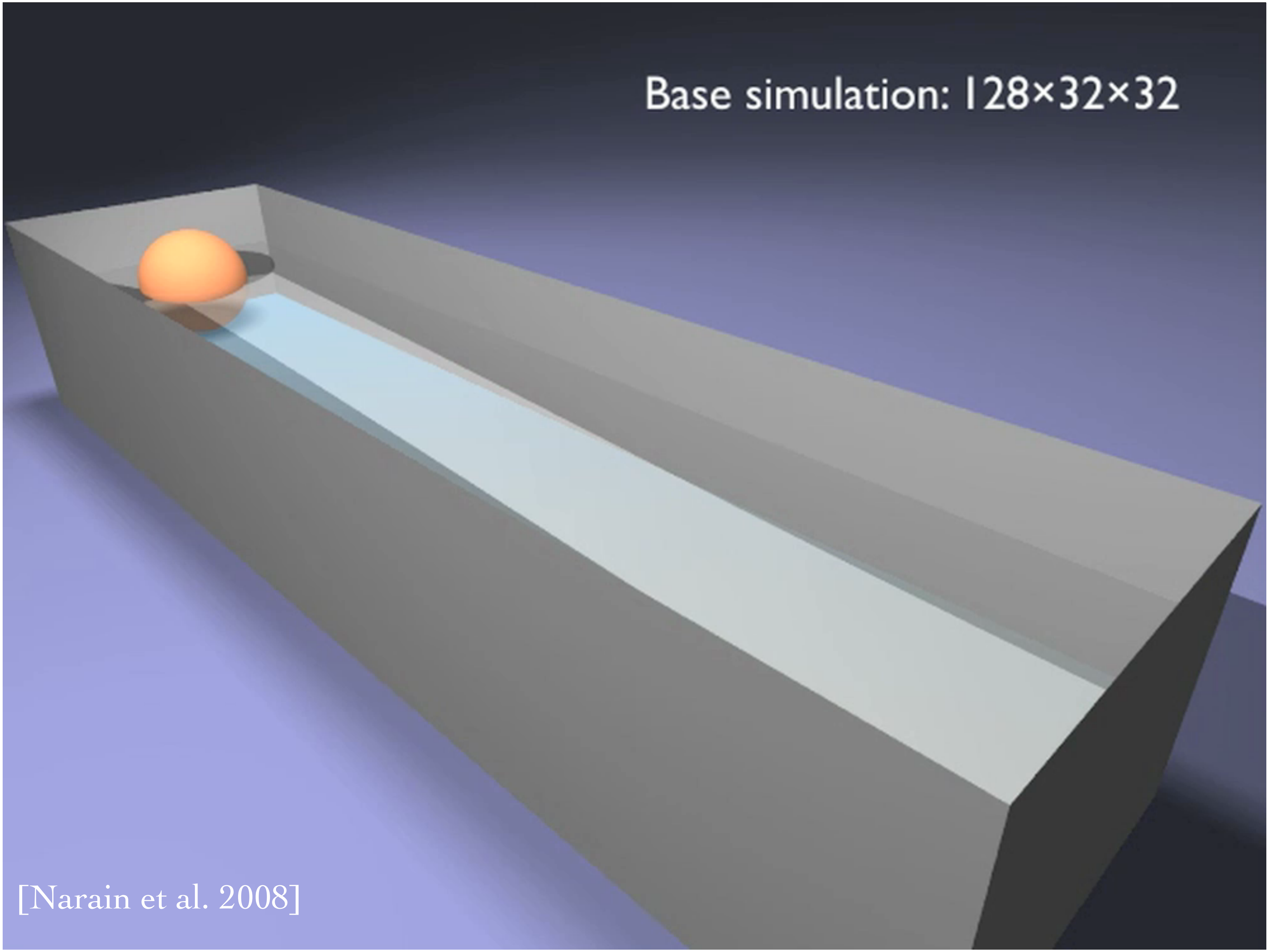
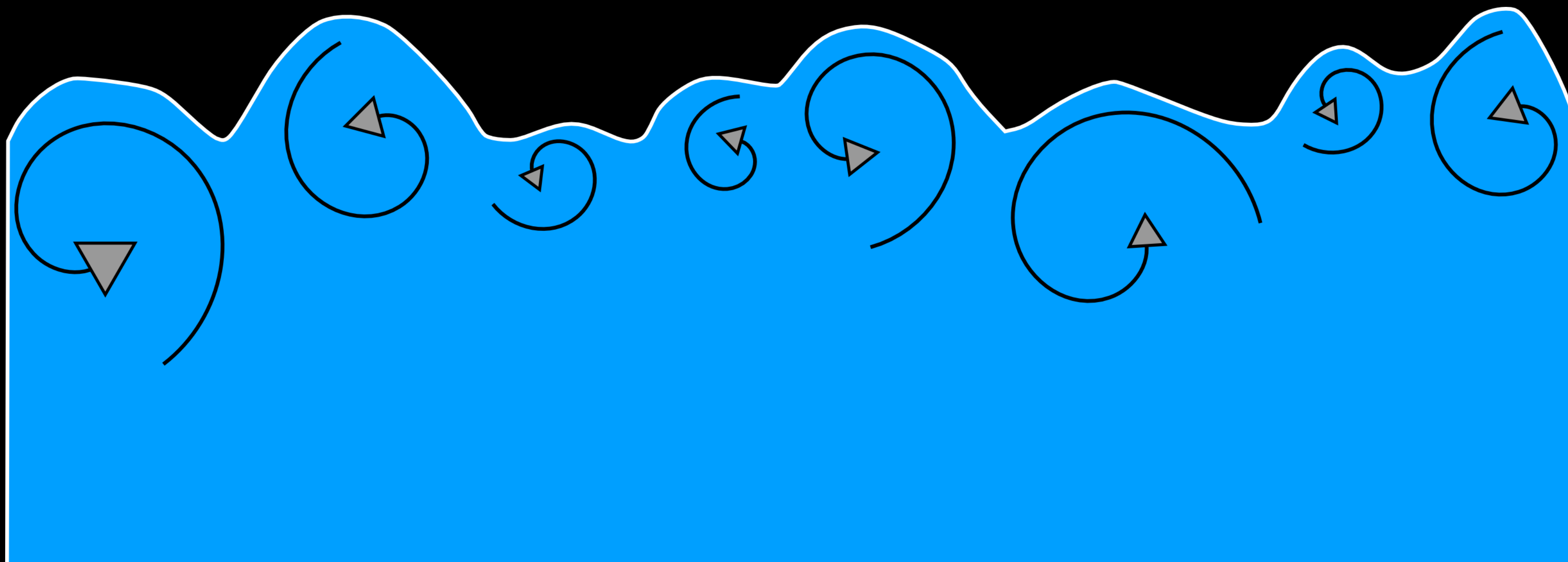

Base simulation: 128×32×32
[Narain et al. 2008]

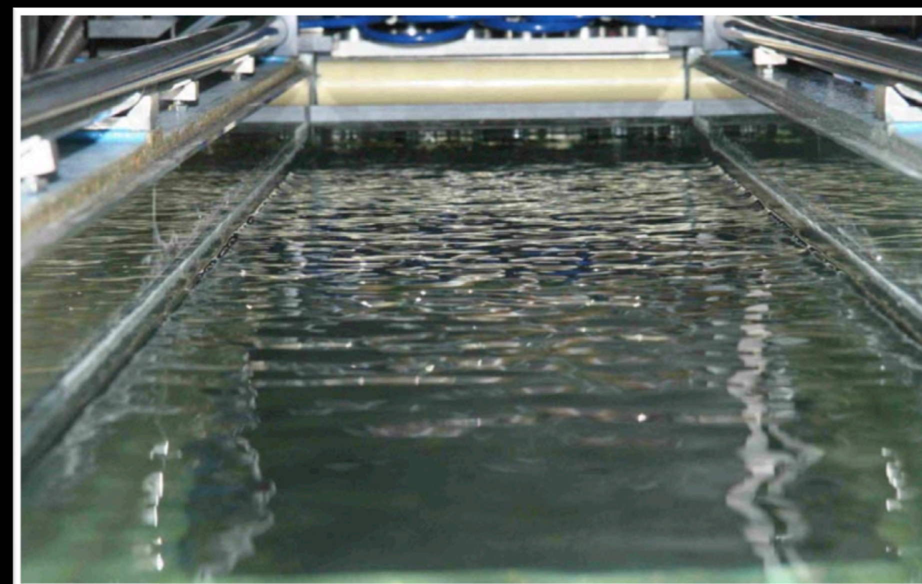
[Pfaff et al. 2009]

[Yuan et al. 2012]

Passive Free Surfaces



[Falcon 2010]



[Savelsberg and van de Water 2008]

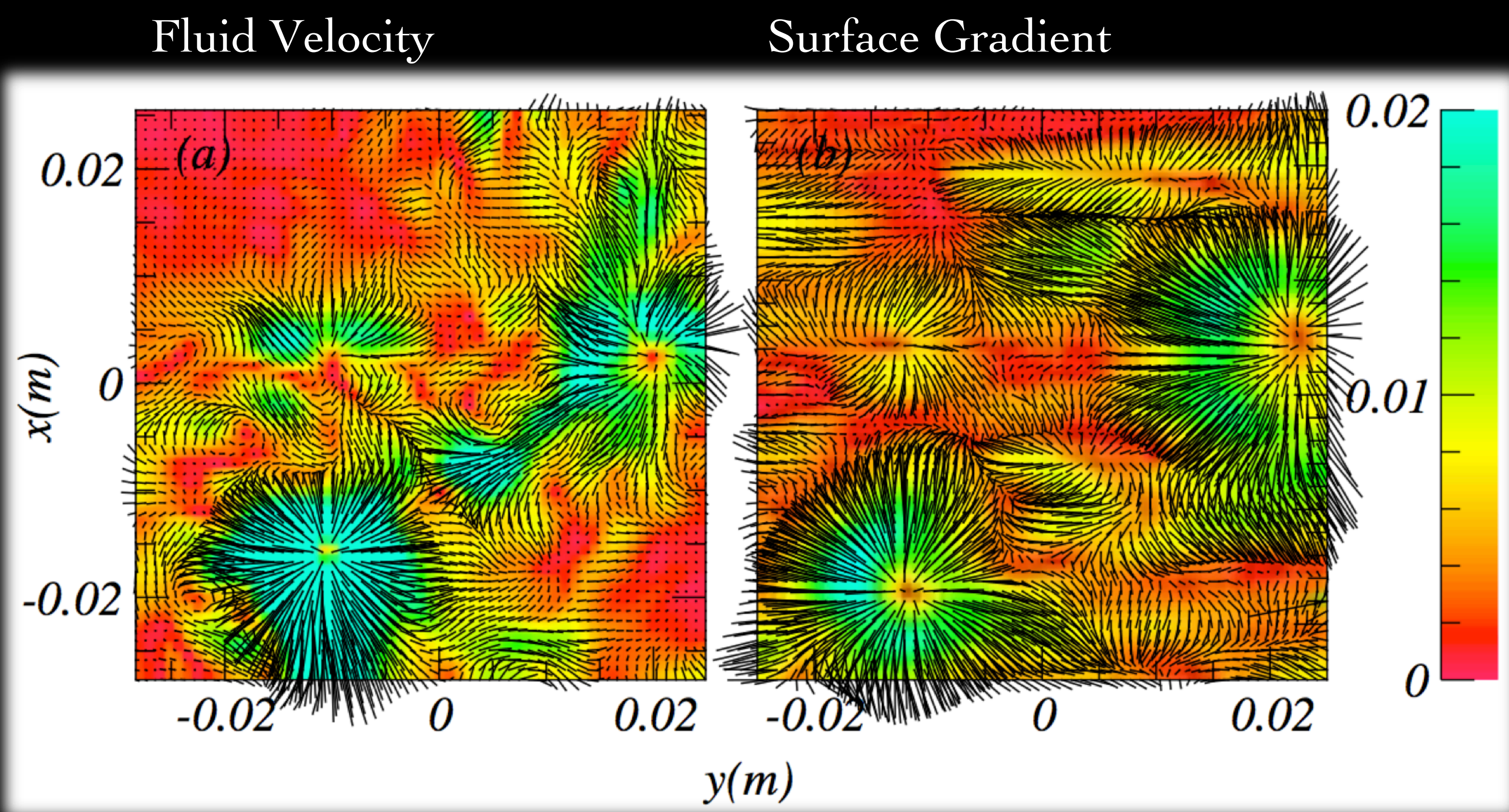


Figure 2, [Savelsberg and van de Water 2008]

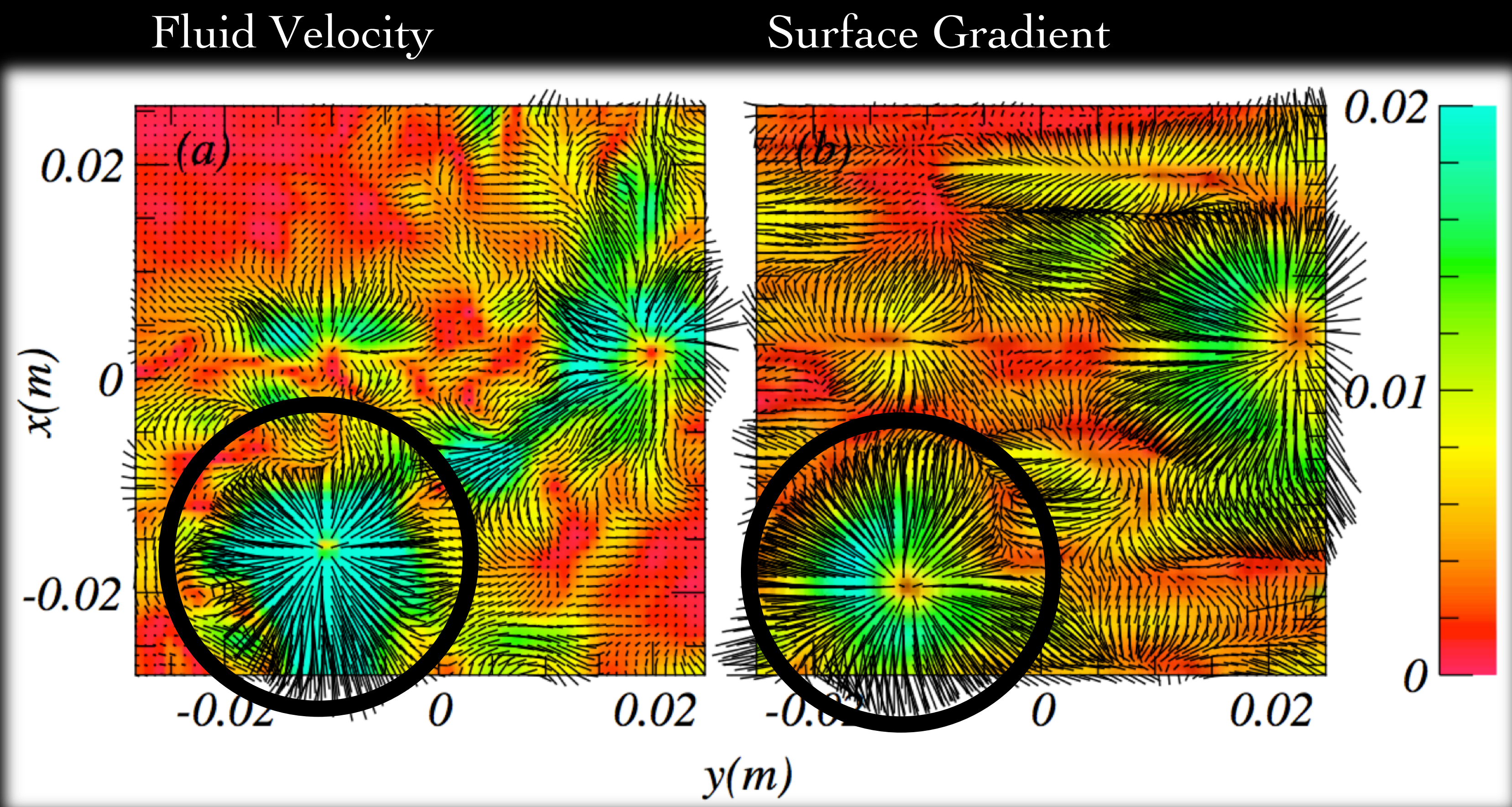


Figure 2, [Savelsberg and van de Water 2008]

Fluid Velocity

Surface Gradient

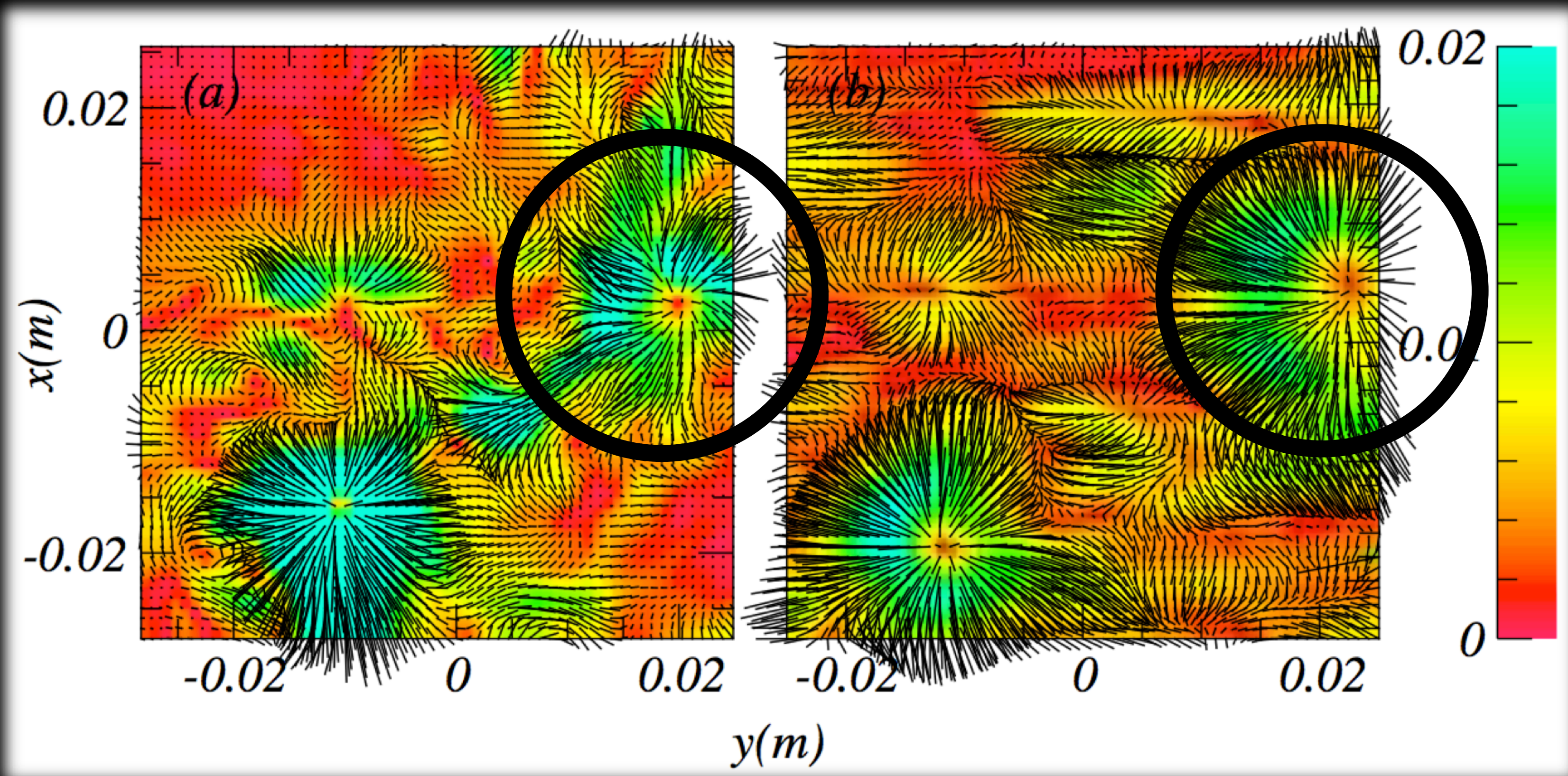


Figure 2, [Savelsberg and van de Water 2008]

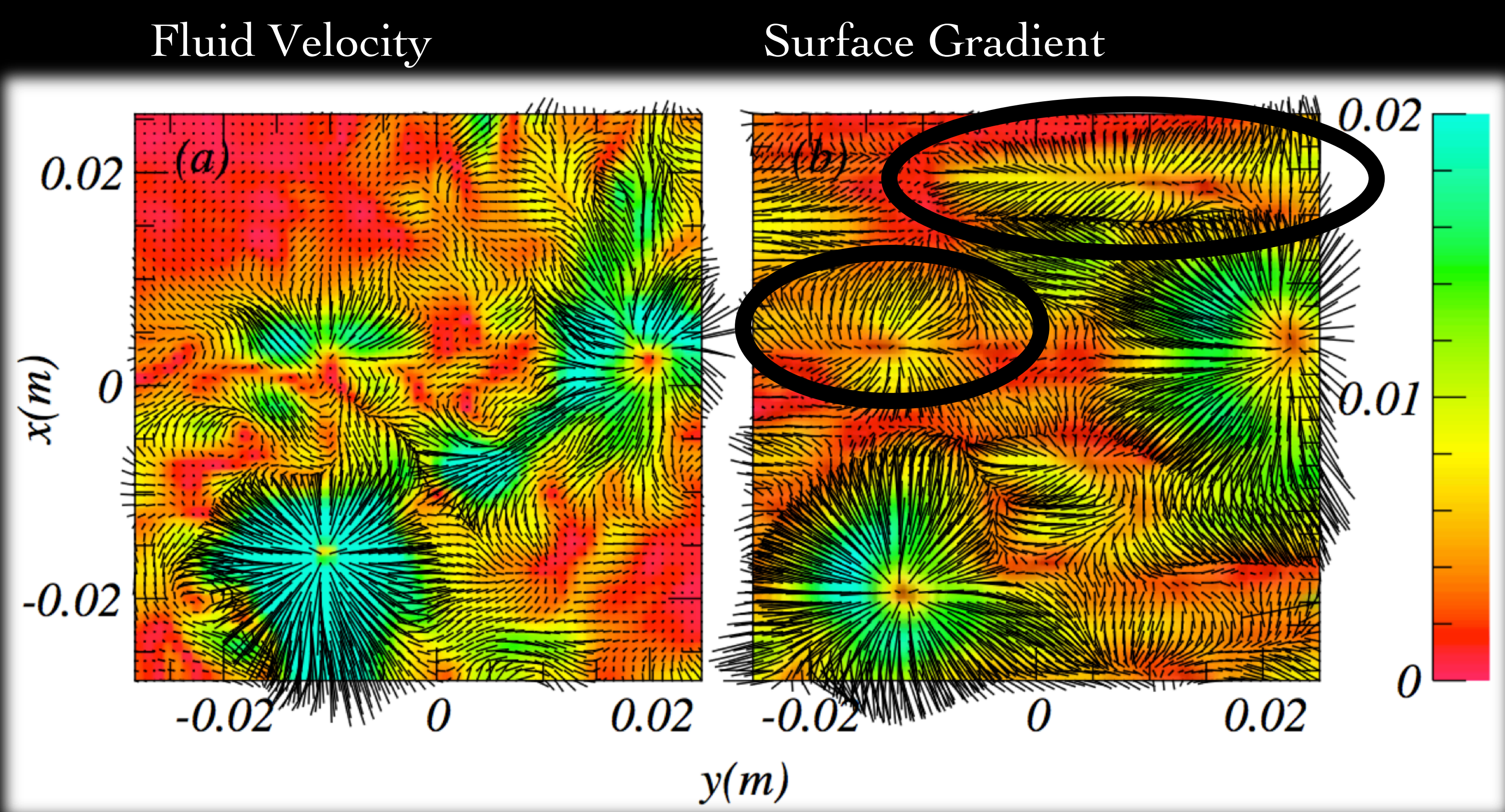


Figure 2, [Savelsberg and van de Water 2008]

Capillary-Gravity Waves

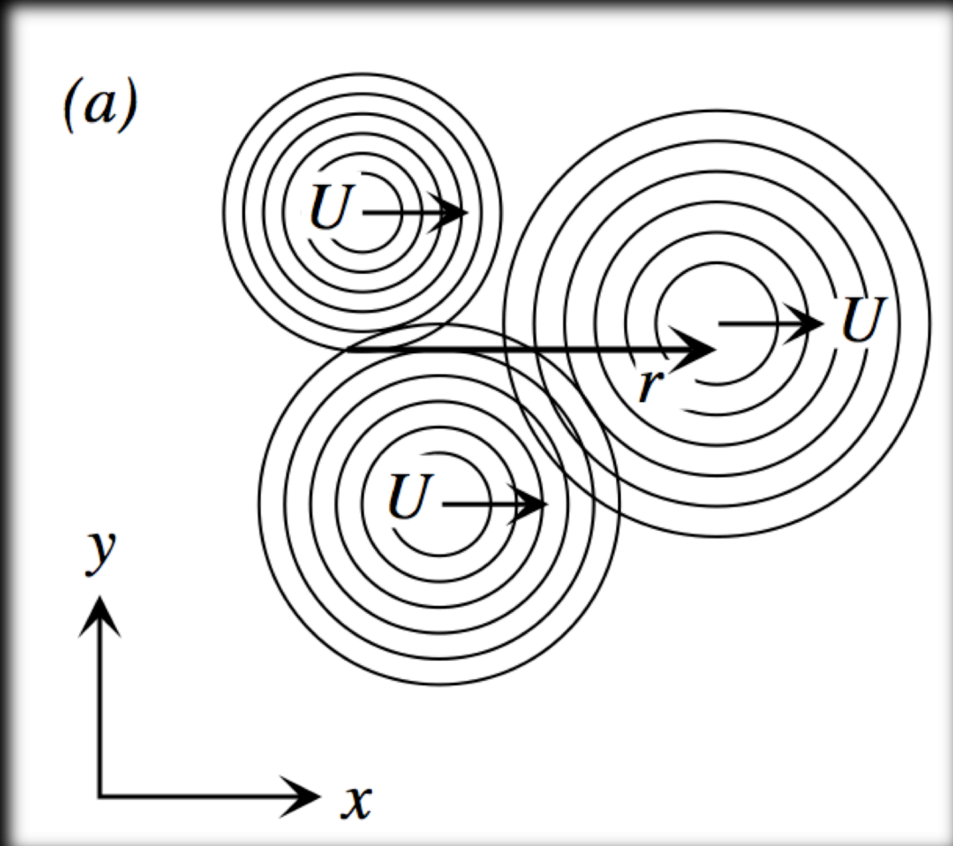
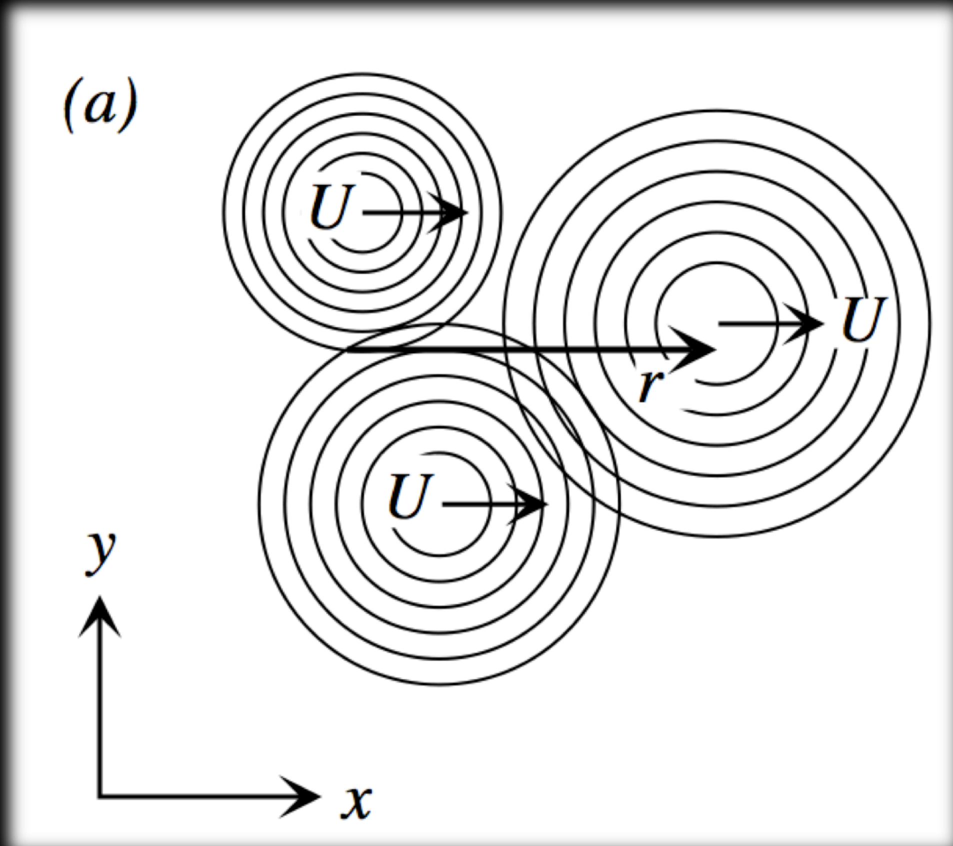


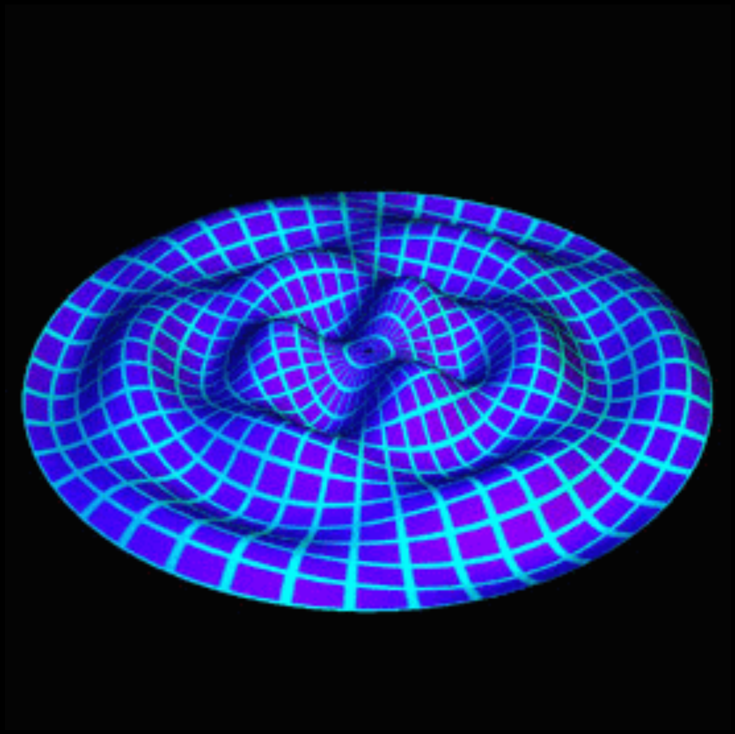
Figure 6, [Savelsberg and van de Water 2008]

Capillary-Gravity Waves



These move *much faster* than the underlying liquid!

Free Surface as a Drumhead



http://www.uh.edu/engines/epi2613.htm

Free Surface Turbulence

- Kolmogorov spectrum: $k^{-\frac{5}{3}}$
- Kolmogorov-*Zakharov* spectrum: $k^{-\frac{11}{4}}$
  - A.k.a. "wave" or "weak" turbulence
  - [Zakharov 1968]
- [Savelbergs and van de Water 2008] found k^{-6}
- [Falcon 2010] describes other exponents

Explicit Wave Simulation

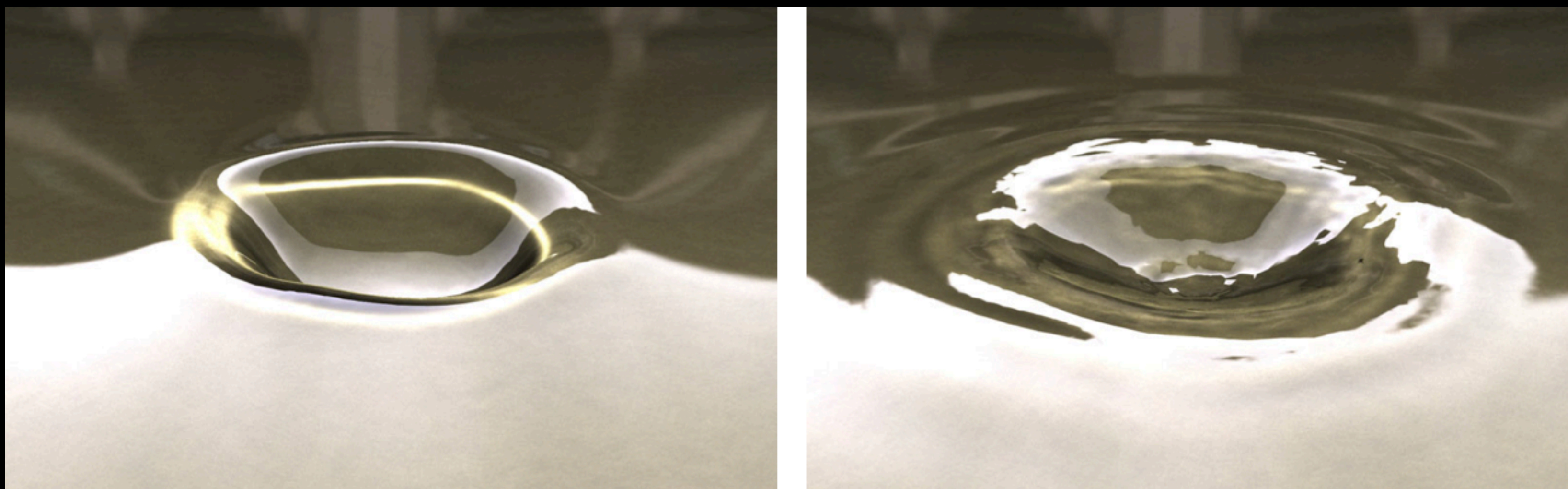


Figure 3, [Thuerey et al. 2010]

Overview

- Related Works
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The Wave Equation

$$\frac{\partial^2 h}{\partial t^2} = c\nabla^2 h$$

The Korteweg-de Vries (KdV) Equation

$$\frac{\partial h}{\partial t} + h\frac{\partial h}{\partial x} + \frac{\partial^3 h}{\partial x^3} = 0$$

The Korteweg-de Vries (KdV) Equation

$$\frac{\partial h}{\partial t} + \boxed{h \frac{\partial h}{\partial x}} + \frac{\partial^3 h}{\partial x^3} = 0$$

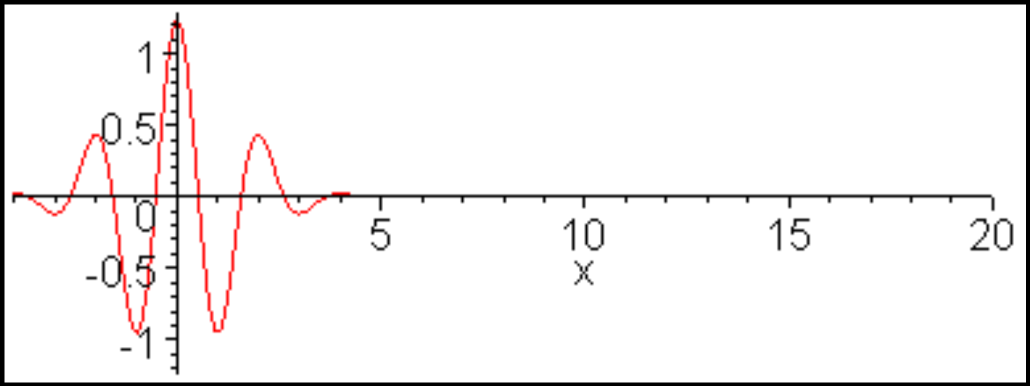
Non-linear!

The Korteweg-de Vries (KdV) Equation

$$\frac{\partial h}{\partial t} + h\frac{\partial h}{\partial x} + \frac{\partial^3 h}{\partial x^3} = 0$$

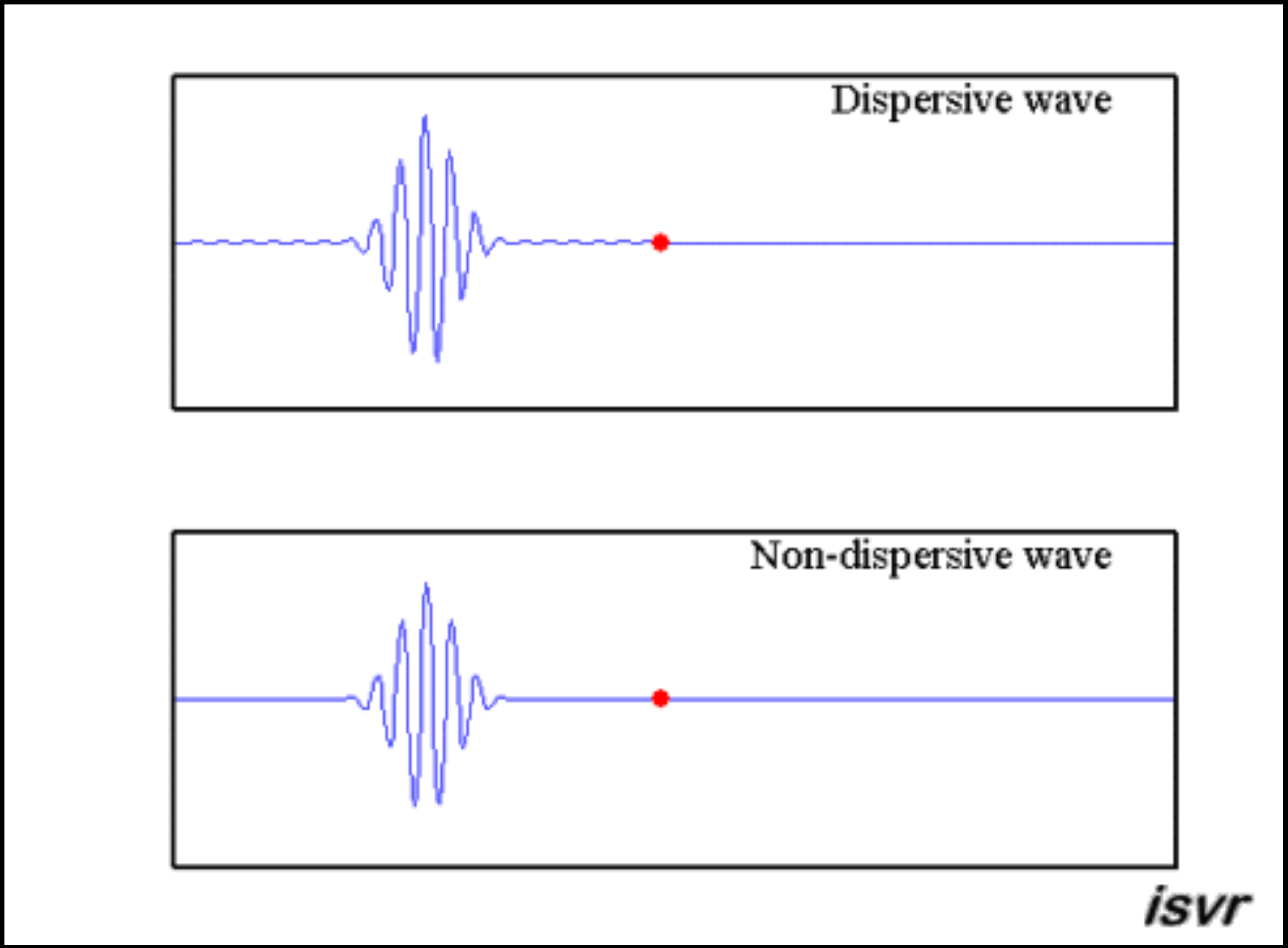
Dispersion

The Korteweg-de Vries (KdV) Equation



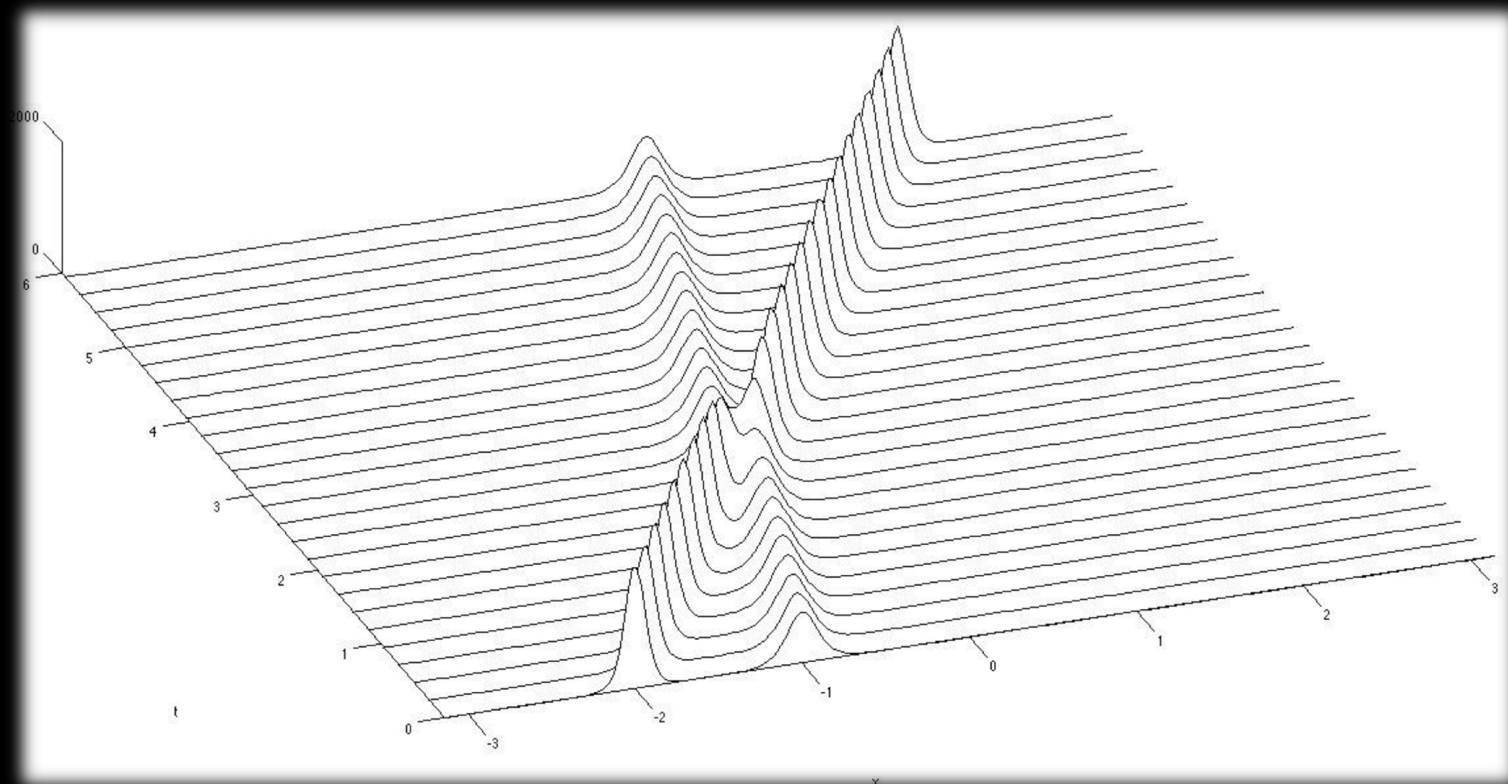
http://physics.usask.ca/~hirose/ep225/animation/dispersion/anim-dispersion.html

The Korteweg-de Vries (KdV) Equation

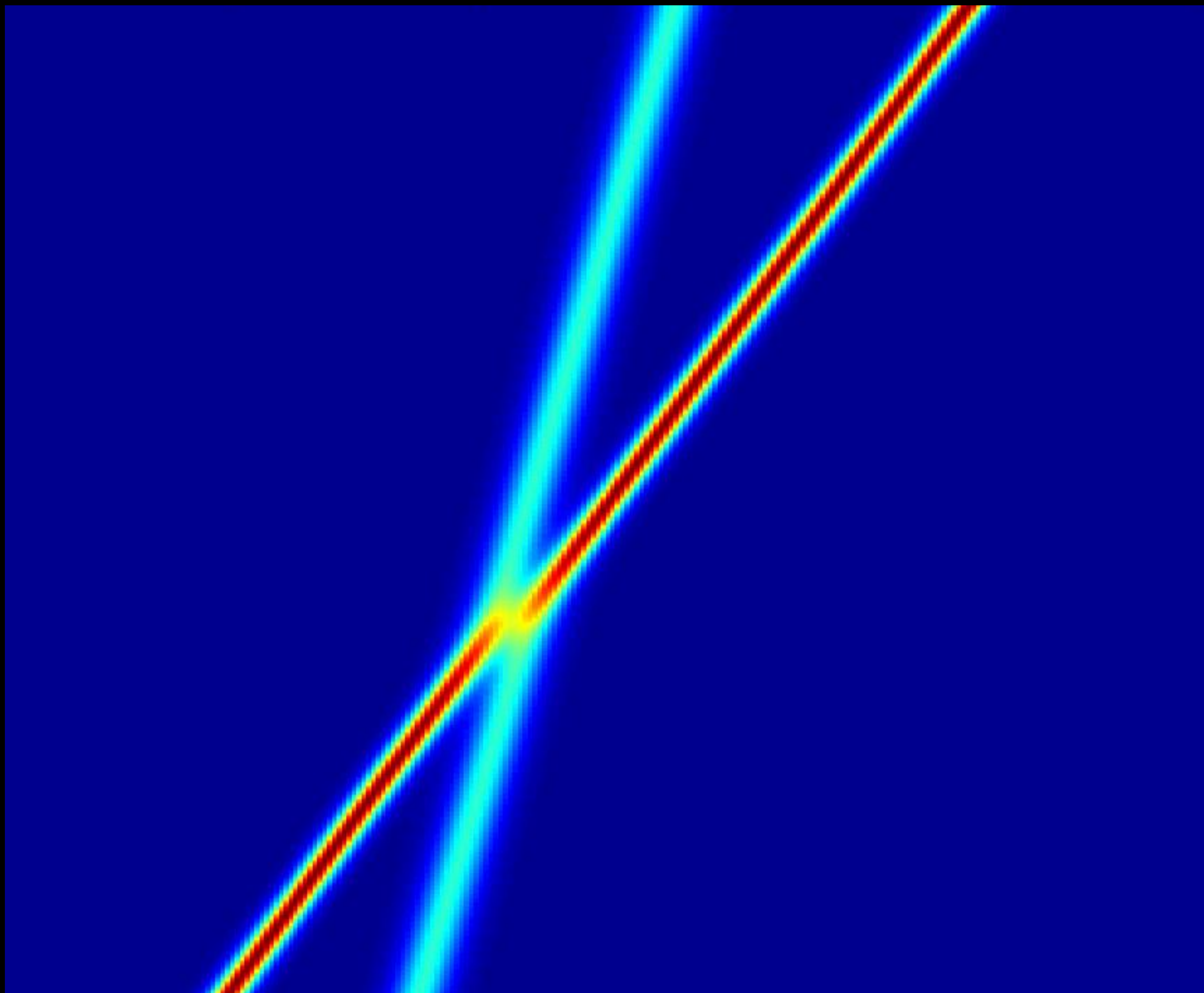


http://resource.isvr.soton.ac.uk/spcg/tutorial/tutorial/Tutorial_files/Web-further-dispersive.htm

The Korteweg-de Vries (KdV) Equation



The Korteweg-de Vries (KdV) Equation



The Kadomtsev-Petviashvili (KP) Equation

$$\frac{\partial}{\partial x}\left(\frac{\partial h}{\partial t} + h\frac{\partial h}{\partial x} + \frac{\partial^3 h}{\partial x^3}\right) + \frac{\partial^2 h}{\partial y^2} = 0$$

The Kadomtsev-Petviashvili (KP) Equation

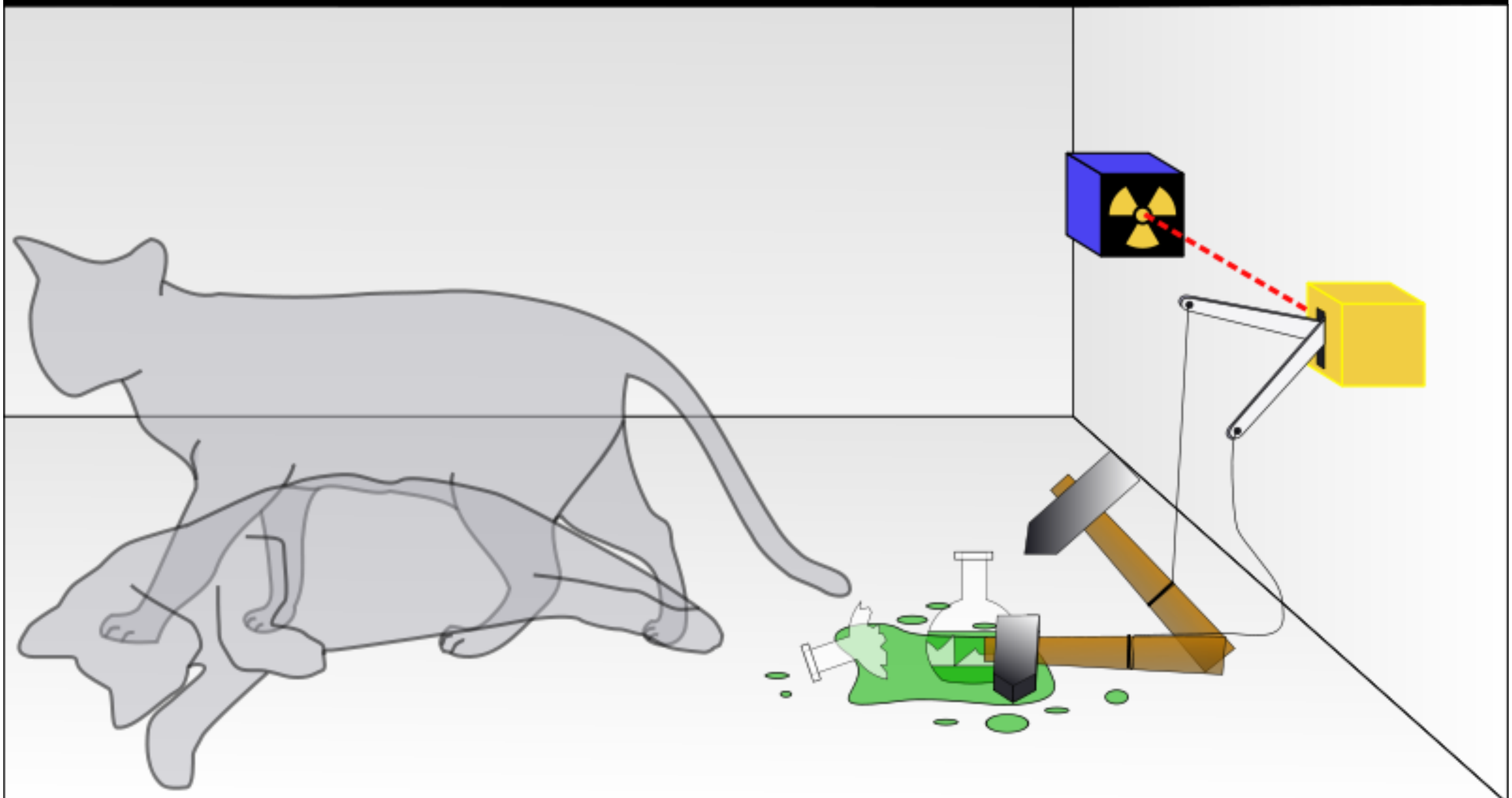
$$\frac{\partial}{\partial x}\left(\frac{\partial h}{\partial t}+h\frac{\partial h}{\partial x}+\boxed{\frac{\partial^3 h}{\partial x^3}}\right)+\boxed{\frac{\partial^2 h}{\partial y^2}}=0$$

Anisotropic!

The Non-Linear Schroedinger Equation

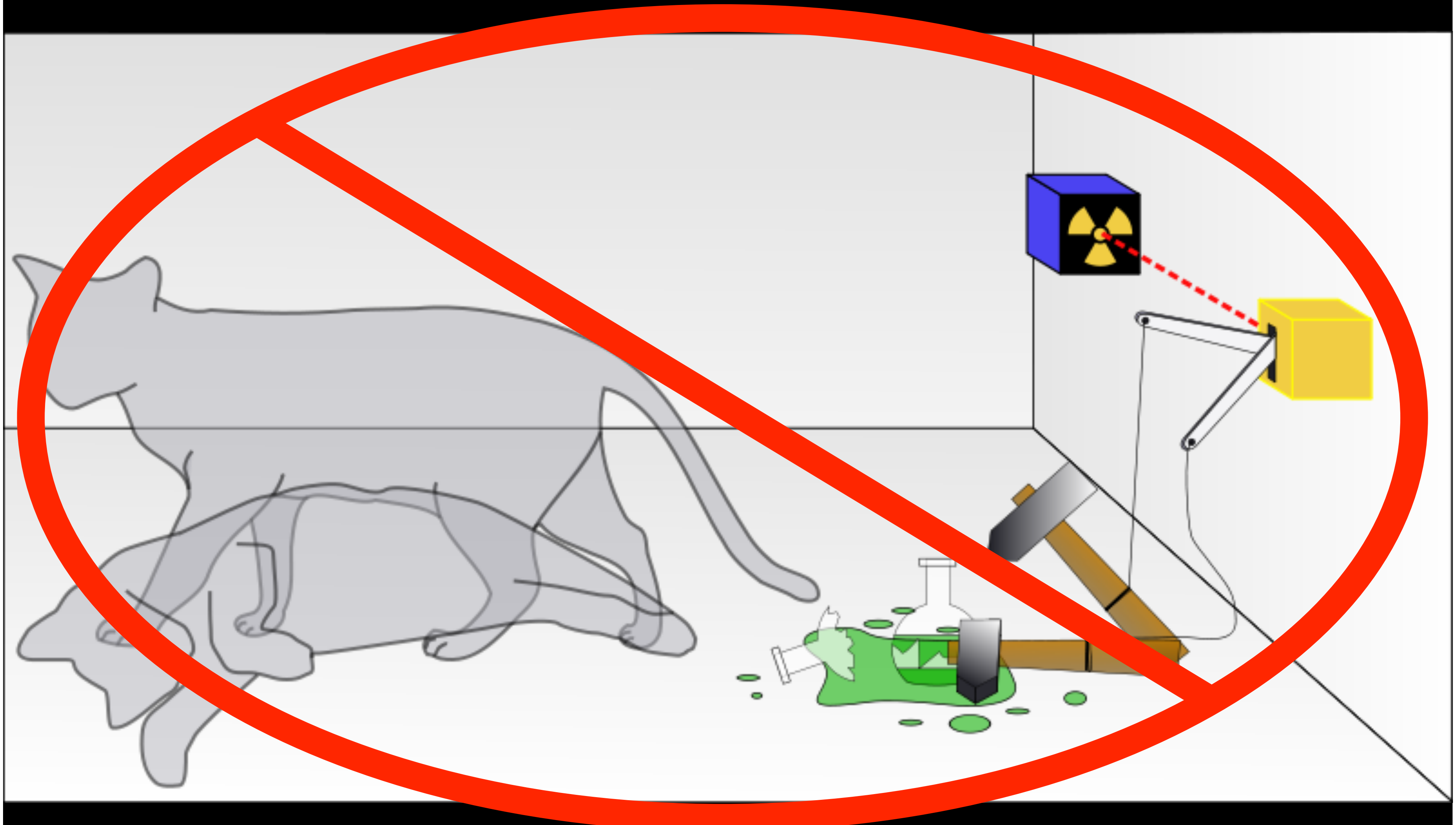
$$i\frac{\partial h}{\partial t} + \alpha \frac{\partial h}{\partial x} = \beta |h|^2 h$$

The Non-Linear Schroedinger Equation



http://www.universetoday.com/54076/schrodingers-cat/

The Non-Linear Schroedinger Equation



http://www.universetoday.com/54076/schrodingers-cat/

The Davey-Stewartson System

$$i\frac{\partial h}{\partial t}+\lambda\frac{\partial^2 h}{\partial x^2}+\frac{\partial^2 h}{\partial y^2}-\upsilon|h|^2 h=h\frac{\partial h}{\partial x}$$

$$\alpha\frac{\partial^2\phi}{\partial x^2}+\frac{\partial^2\phi}{\partial y^2}=\xi\frac{\partial|h|^2}{\partial x}$$

The Davey-Stewartson System

$$i\frac{\partial h}{\partial t}+\lambda\frac{\partial^2 h}{\partial x^2}+\frac{\partial^2 h}{\partial y^2}-\upsilon|h|^2 h=\boxed{h\frac{\partial h}{\partial x}}$$

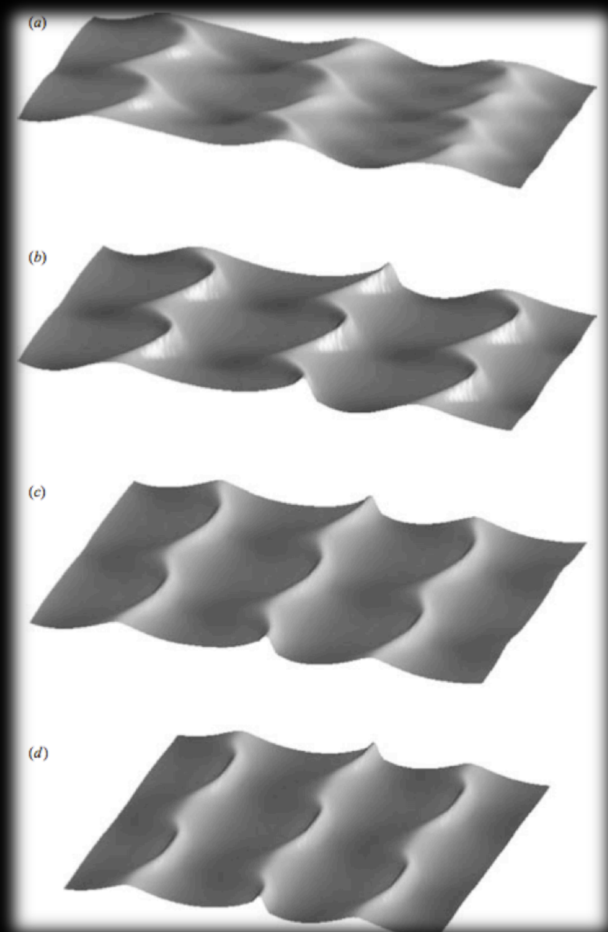
$$\alpha\frac{\partial^2 \phi}{\partial x^2}+\frac{\partial^2 \phi}{\partial y^2}=\boxed{\xi\frac{\partial |h|^2}{\partial x}}$$

Anisotropic!

What are the good for?

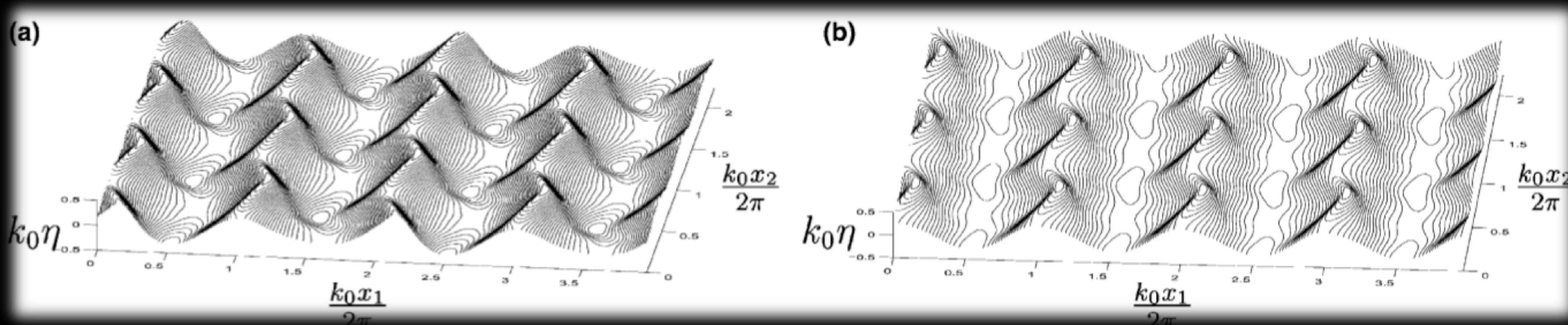
- The Kadomtsev-Petviashvili (KP) Equation
- The Davey-Stewartson System

Horseshoe Waves



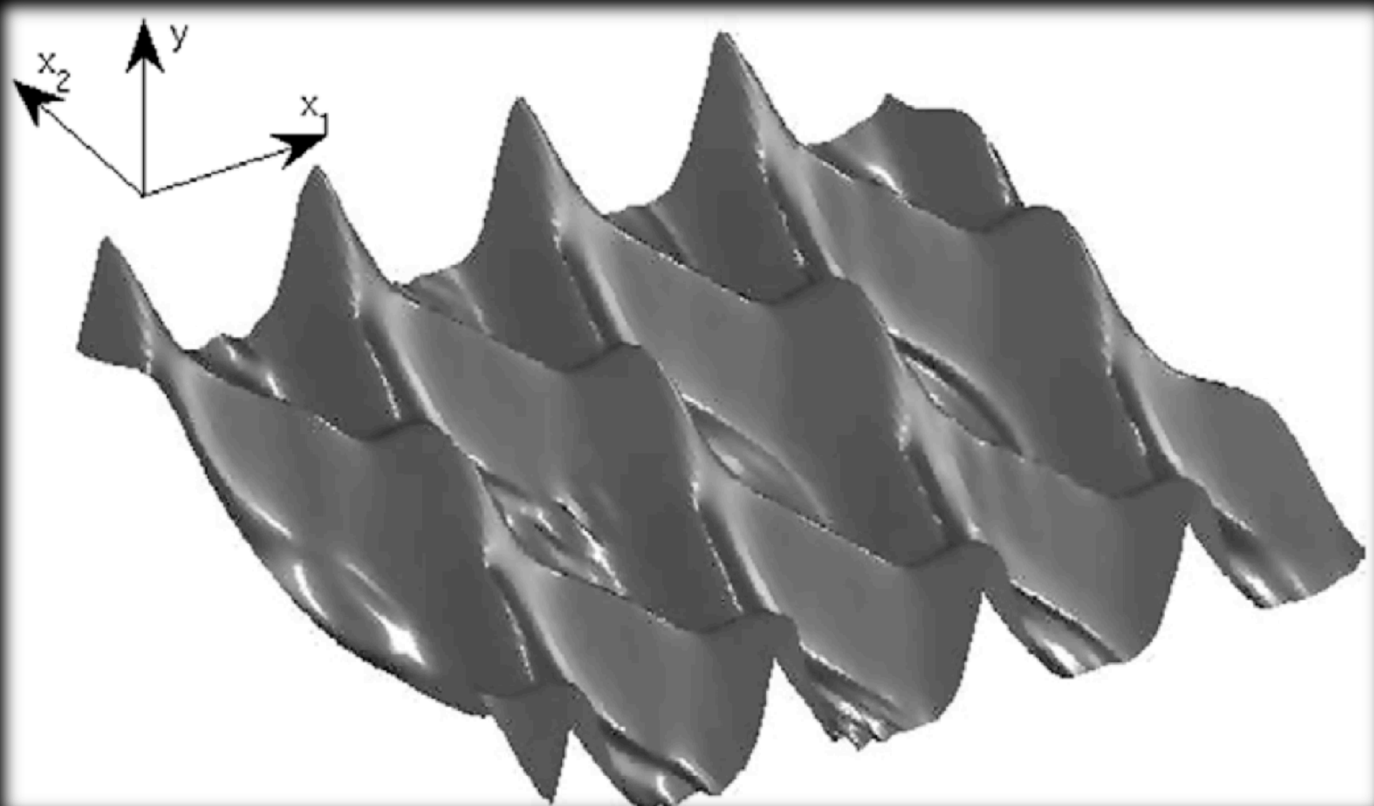
[Fuhrman et al. 2004]

Horseshoe Waves



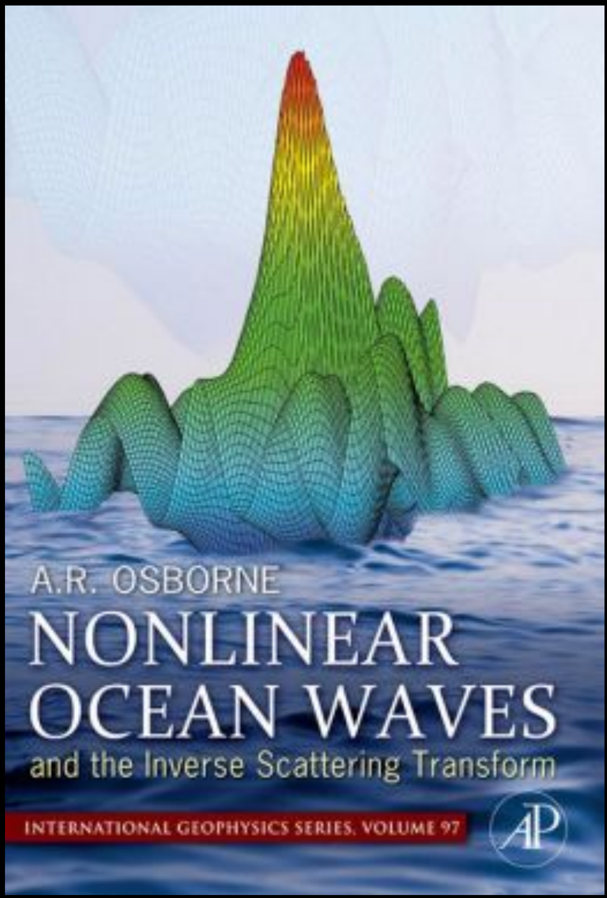
[Fructus et al. 2005]

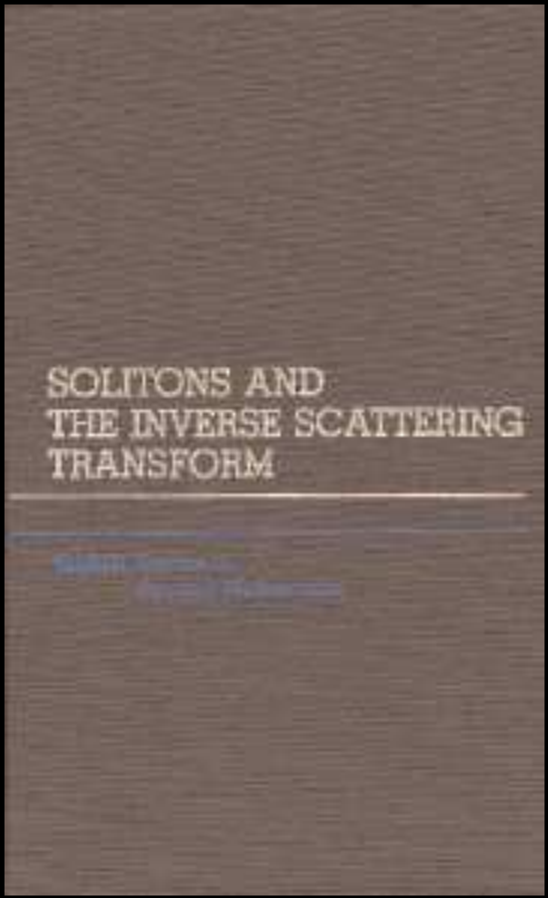
HexagonalWaves

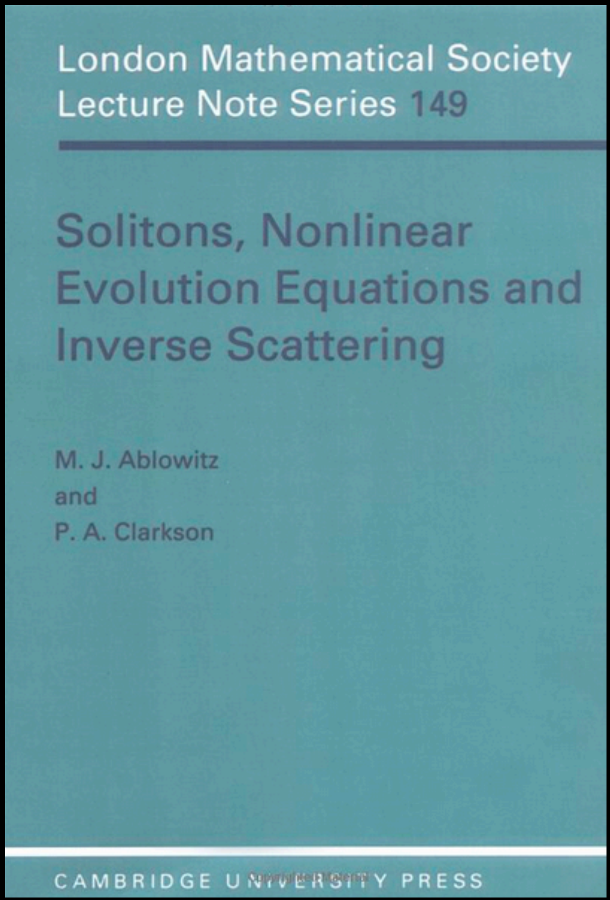


[Xu et al. 2009]

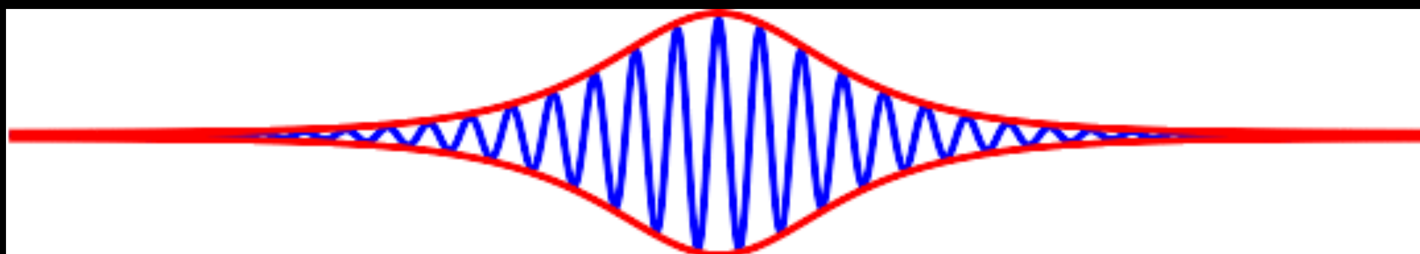
Inverse Scattering Transform







Solitons



The Korteweg-de Vries (KdV) Equation

$$\frac{\partial h}{\partial t} + h\frac{\partial h}{\partial x} + \frac{\partial^3 h}{\partial x^3} = 0$$

- Dispersion
- Phase shifting

The Wave Equation

$$\frac{\partial^2 h}{\partial t^2} = c\nabla^2 h$$

- ~~Dispersion~~
- ~~Phase shifting~~

The iWave Equation [Tessendorf 2004]

$$\frac{\partial^2 h}{\partial t^2} = c\sqrt{-\nabla^2}\, h$$

- Dispersion
- ~~Phase shifting~~

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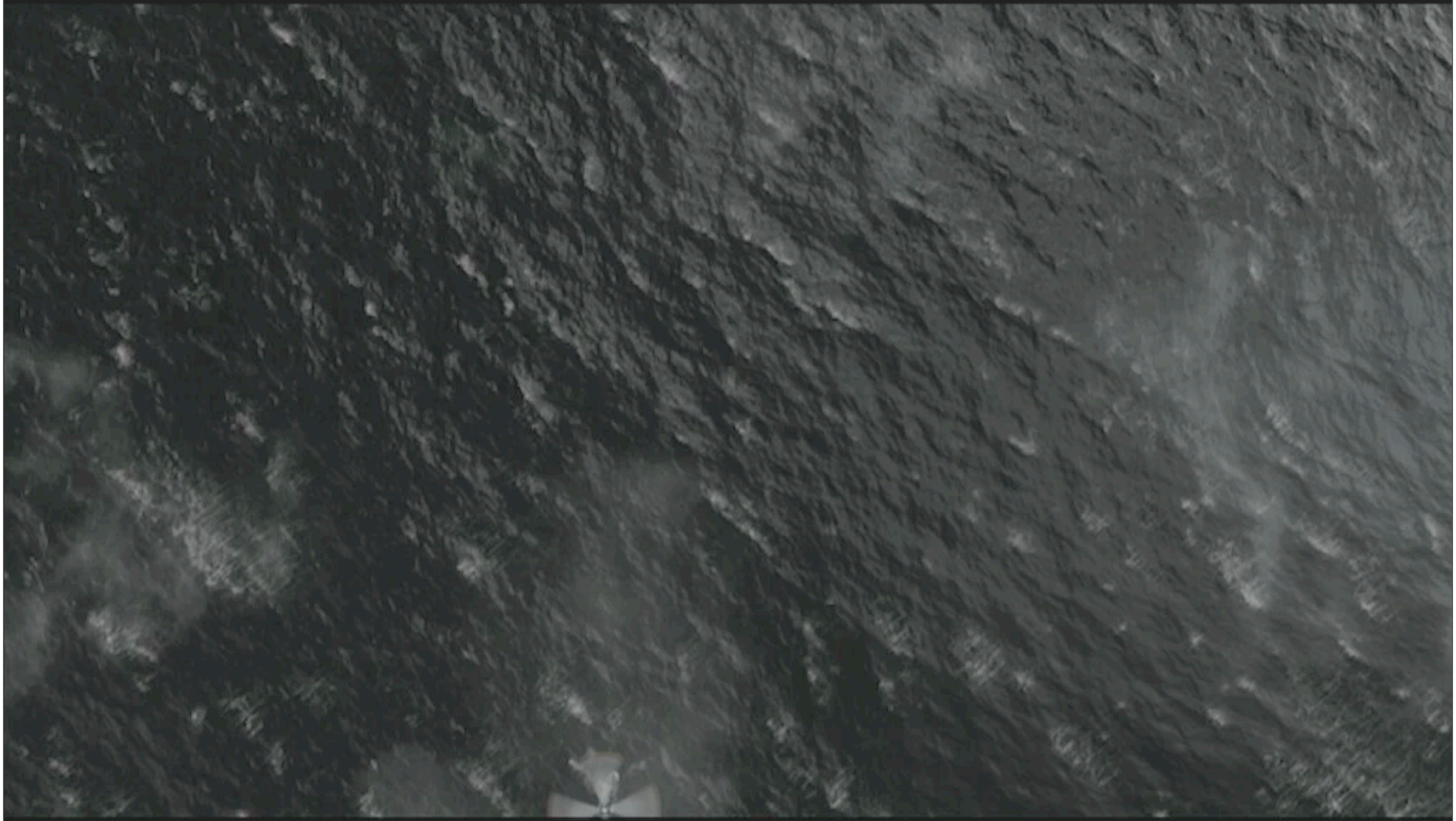
Superman Returns (2006)



Surf’s Up (2007)

Happy Feet (2006)

1163 #653354 : user:slo sc49.08:CmpMain.Main-0046 - 15:03 Oct 02 iWave Equation:

$$\frac{\partial^2 h}{\partial t^2} = -\sqrt{\nabla^2}\, h$$

Wave Equation:

$$\frac{\partial^2 h}{\partial t^2} = c\nabla^2 h$$

ϕ is a velocity potential

ϕ is a velocity potential

$$\nabla\phi = \mathbf{u}$$

ϕ is a velocity potential

$$\nabla\phi = \mathbf{u}$$

$$\frac{\partial h}{\partial t} = \frac{\partial \phi}{\partial y}$$

$$\nabla^2 \phi = 0$$

$$\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}+\frac{\partial^2}{\partial z^2}\right)\phi = 0$$

$$\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}\right)\phi = 0$$

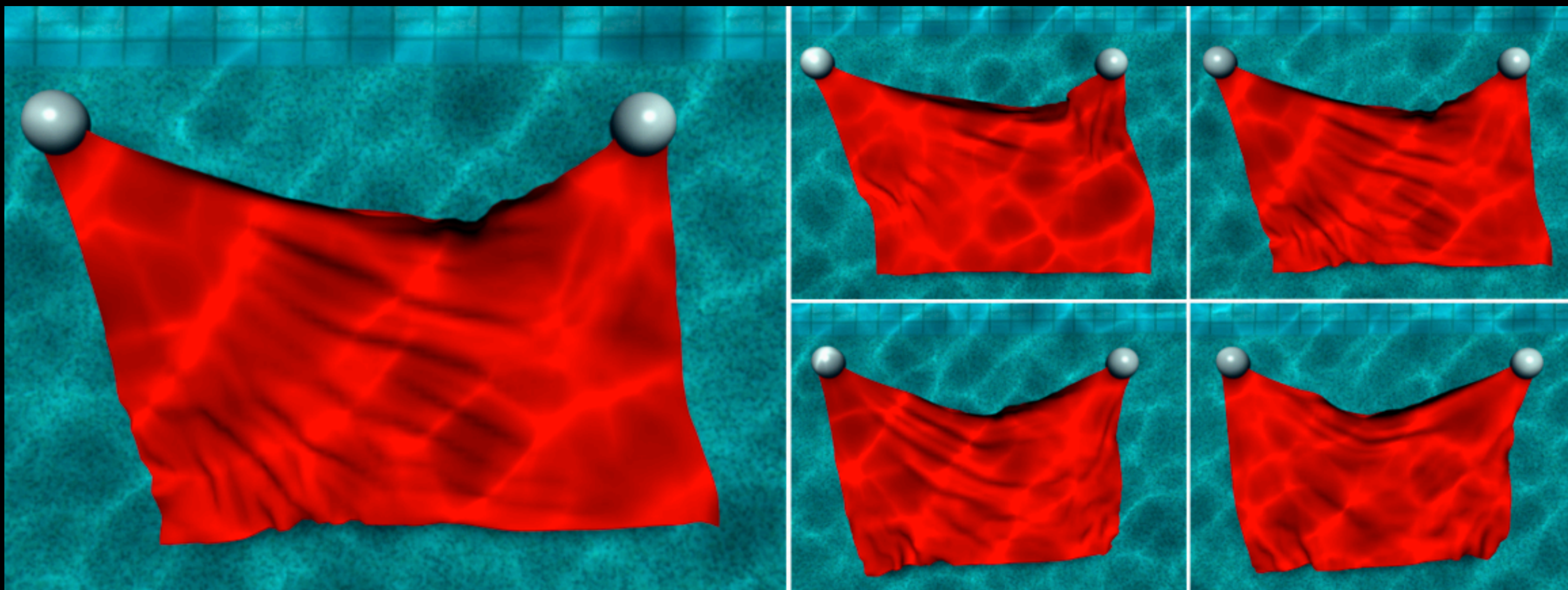
$$\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial z^2}\right)=\nabla_\perp^2$$

$$\left(\nabla_\perp^2+\frac{\partial^2}{\partial y^2}\right)\phi=0$$

$$-\nabla_{\perp}^{2}\phi = \frac{\partial^{2}\phi}{\partial y^{2}}$$

$$\sqrt{-\nabla_{\perp}^{2}\phi} = \sqrt{\frac{\partial^{2}\phi}{\partial y^{2}}}$$

$$\sqrt{-\nabla_{\perp}^{2}}\,\phi = \frac{\partial \phi}{\partial y}$$



[Ozgen et al. 2010]

$$\sqrt{-\nabla_{\perp}^{2}}\,\phi = \frac{\partial \phi}{\partial y}$$

$$\frac{\partial h}{\partial t} = \frac{\partial \phi}{\partial y}$$

$$\sqrt{-\nabla_{\perp}^{2}}\,\phi = \frac{\partial h}{\partial t}$$

$$\sqrt{-\nabla_{\perp}^{2}}\,\phi = \frac{\partial h}{\partial t}$$

$$\sqrt{-\nabla_{\perp}^{2}}\,\frac{\partial \phi}{\partial t}=\frac{\partial^{2} h}{\partial t^{2}}$$

$$\frac{\partial \phi}{\partial t} = ch$$

$$\sqrt{-\nabla_{\perp}^{2}}\,\frac{\partial \phi}{\partial t} = \frac{\partial^{2} h}{\partial t^{2}}$$

$$c\sqrt{-\nabla_{\perp}^{2}}\,h = \frac{\partial^{2} h}{\partial t^{2}}$$

$$\nabla^2 \phi$$

	1	
1	-4	1
	1	

$$\sqrt{\nabla^2}\phi$$

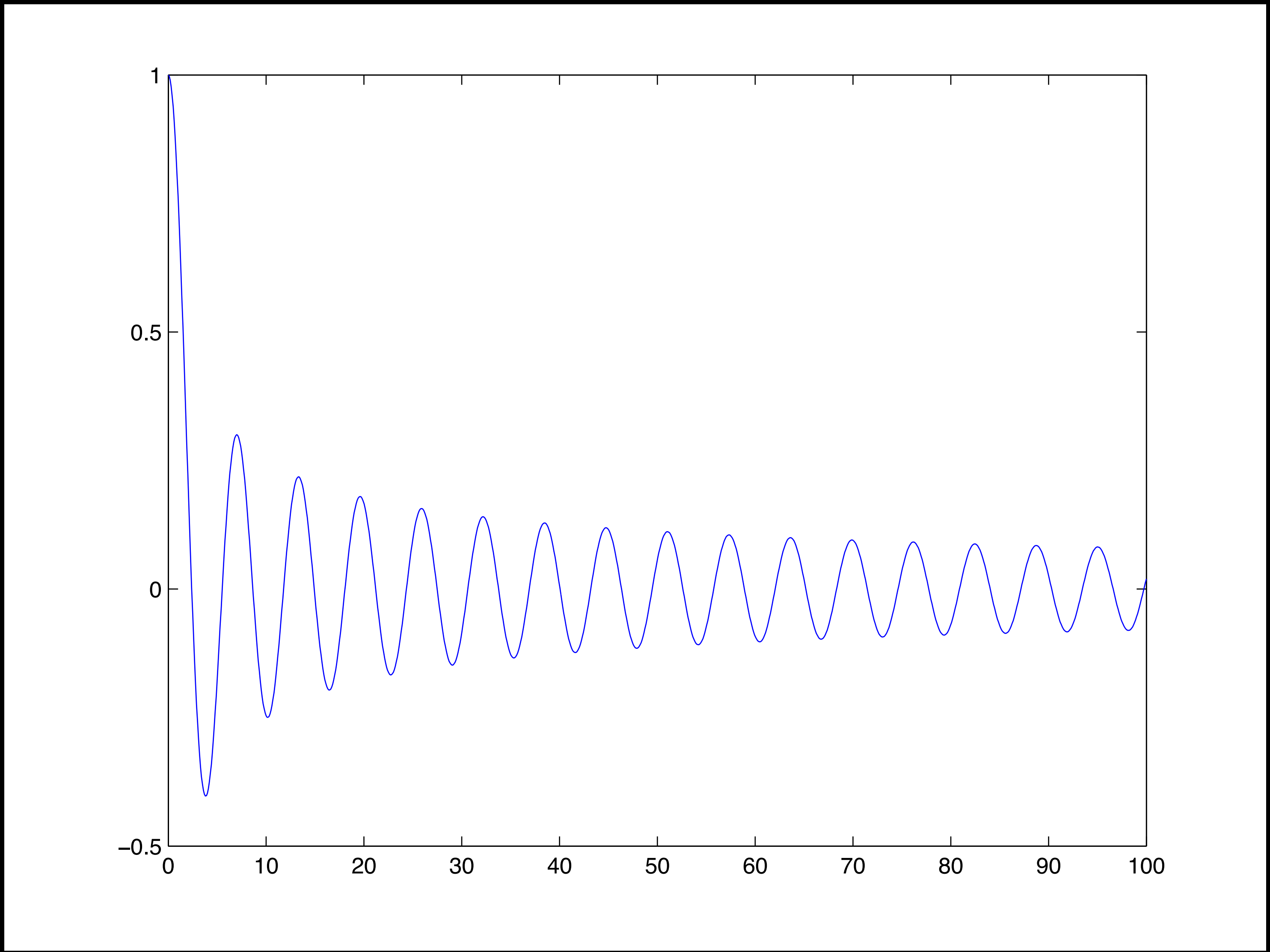
????

$$\sqrt{\nabla^2}\phi \quad \Rightarrow \quad G(\mathbf{x}) = \int_0^\infty k^2 e^{-k^2\sigma^2} J_0(k|\mathbf{x}|)\,dk$$

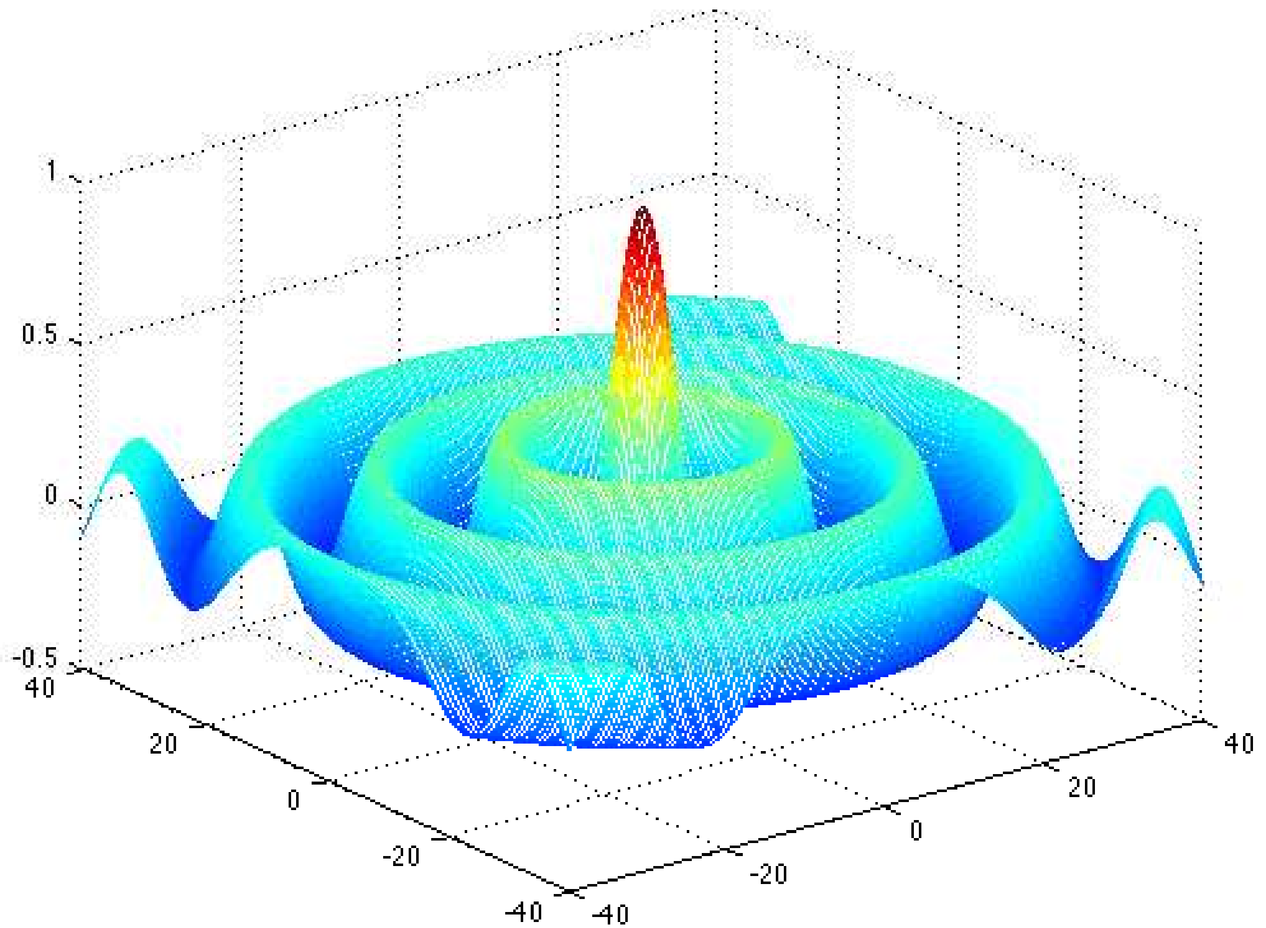
$$G(\mathbf{x}) = \int_0^\infty k^2 e^{-k^2\sigma^2} J_0(k|\mathbf{x}|)\,dk$$

$$G(\mathbf{x}) = \int_0^\infty J_0(k|\mathbf{x}|)\,dk$$

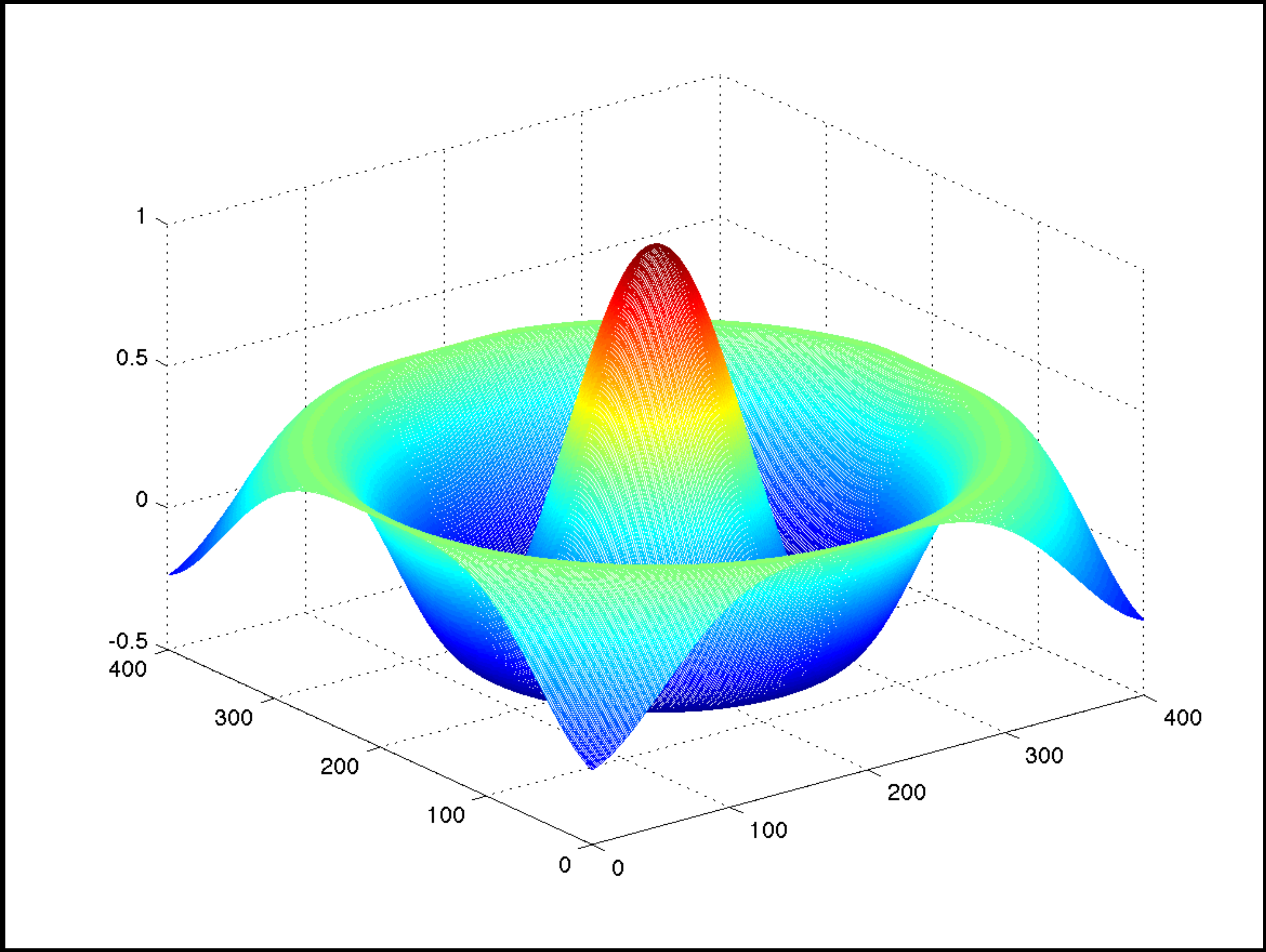
$J_0(x)$



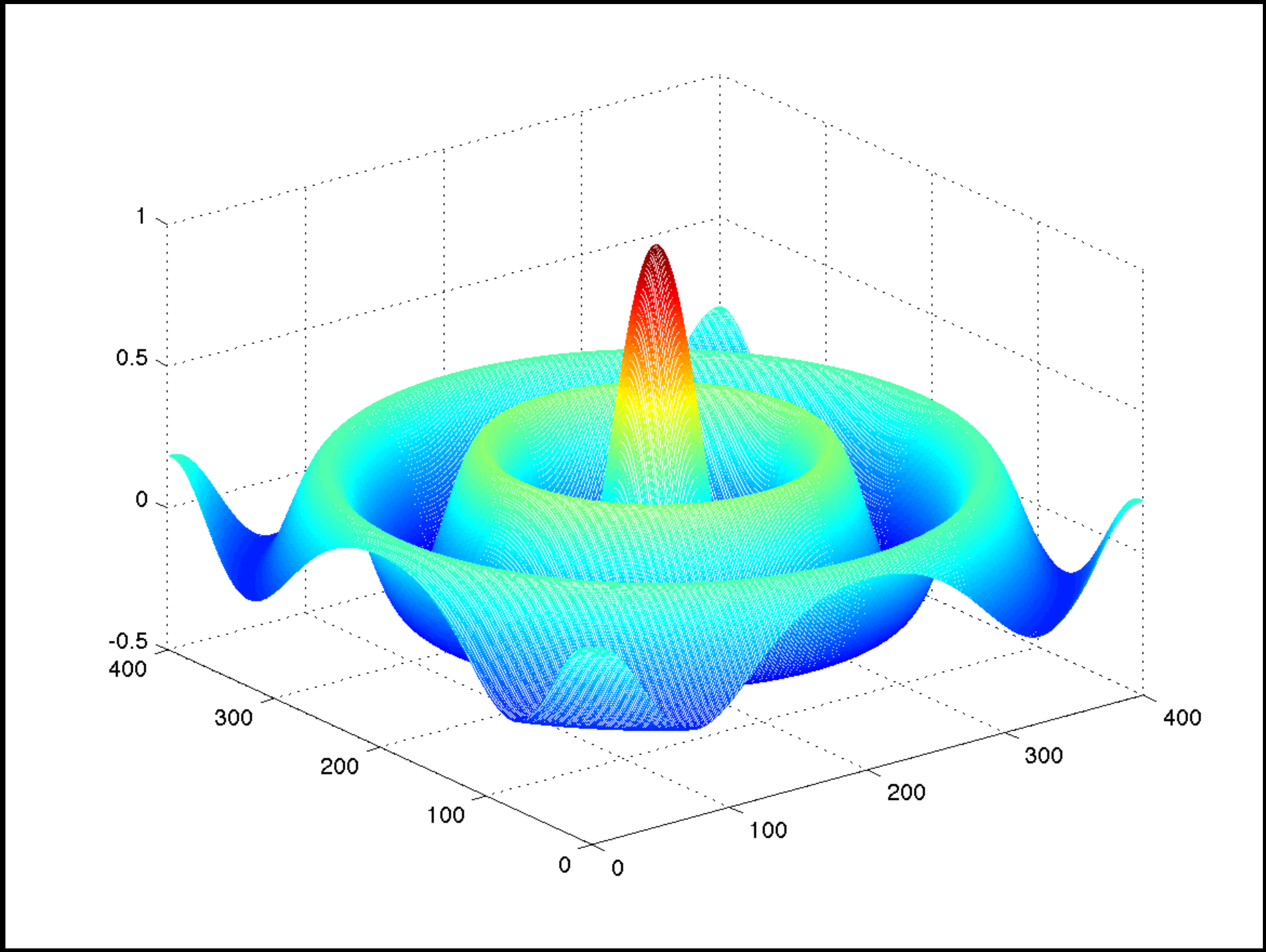
$$J_0(|\mathbf{x}|)$$



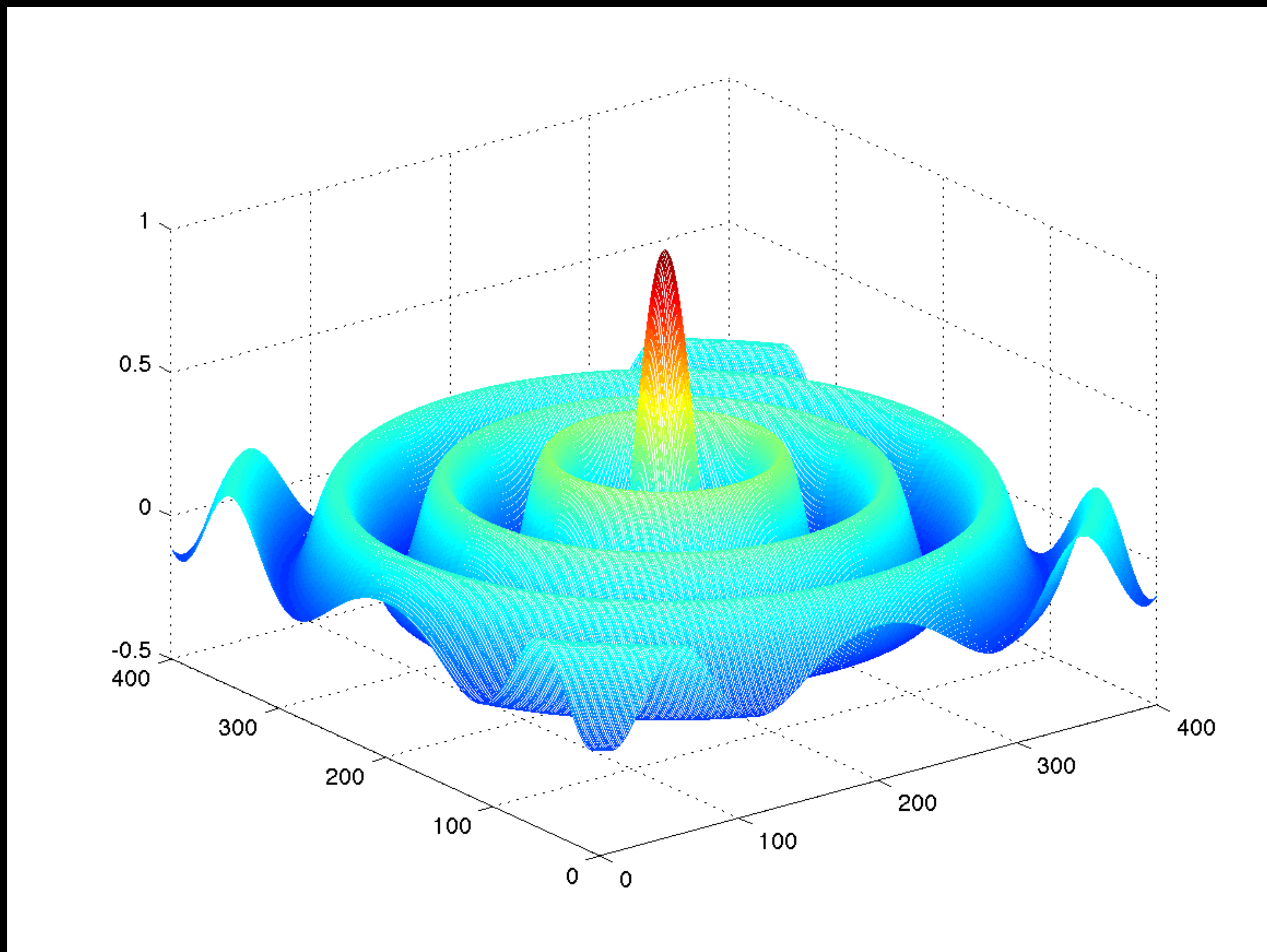
$$J_0(k|\mathbf{x}|), k = 1$$



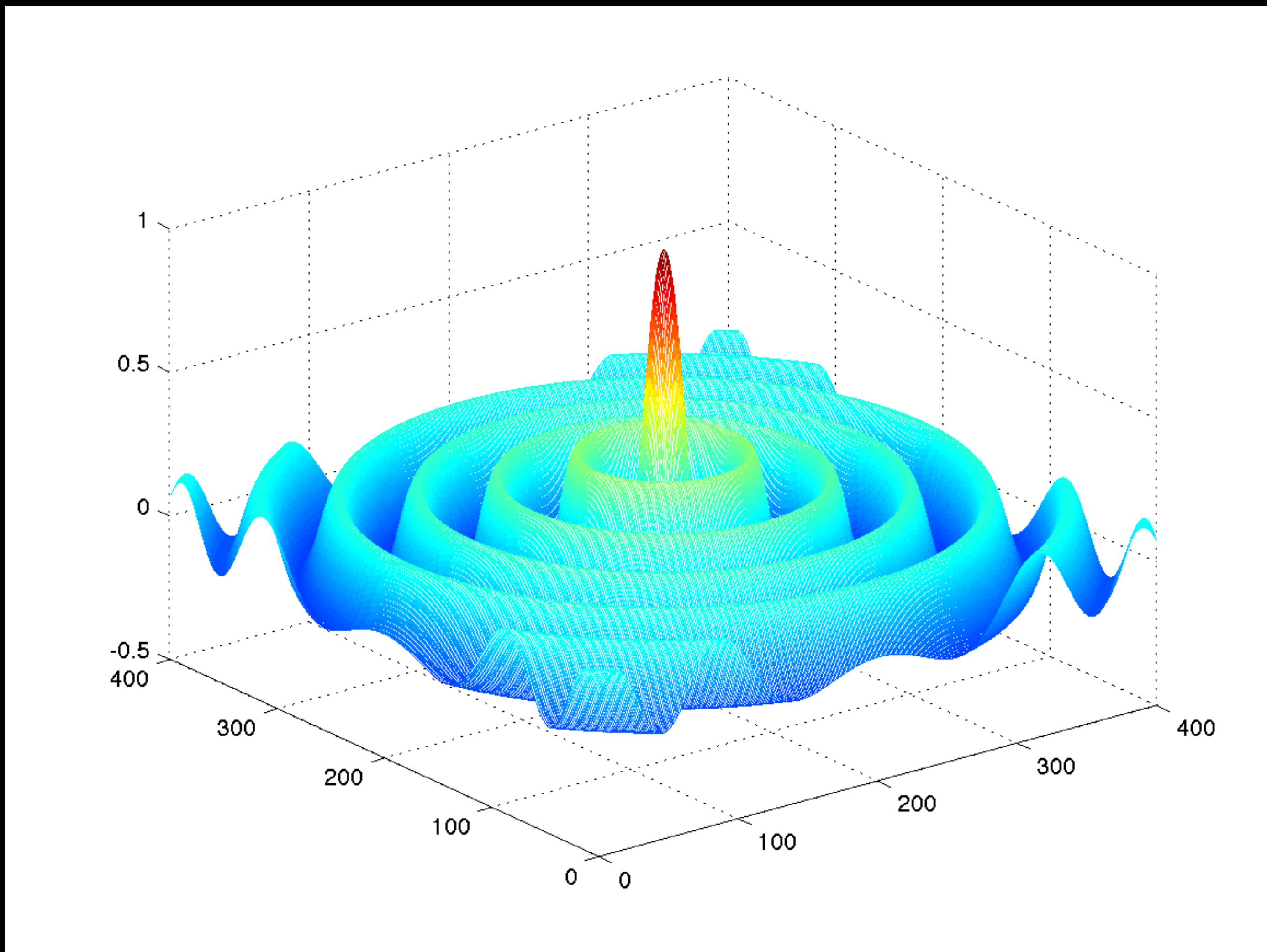
$$J_0(k|\mathbf{x}|), k = 2$$



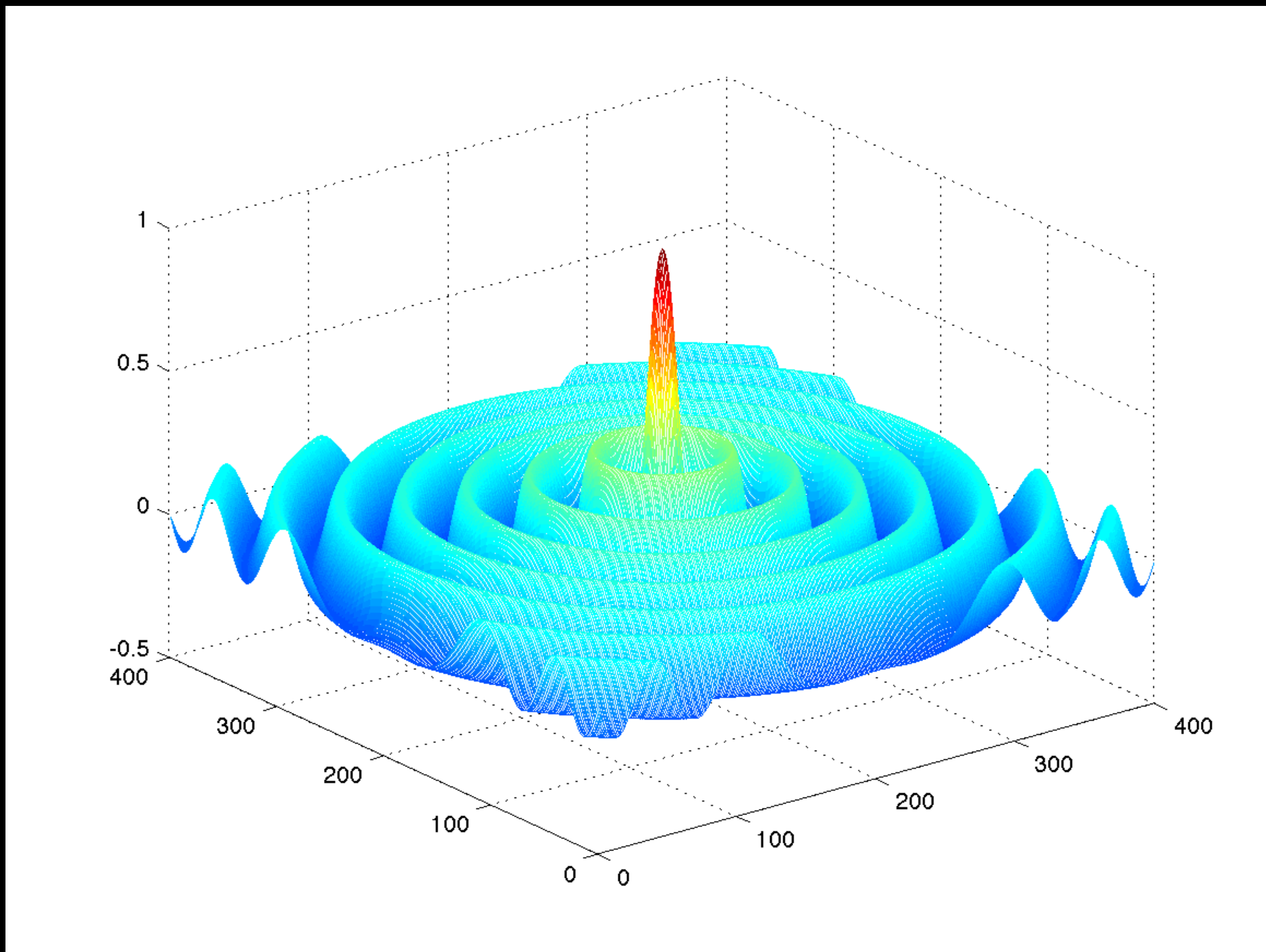
$$J_0(k|\mathbf{x}|), k = 3$$



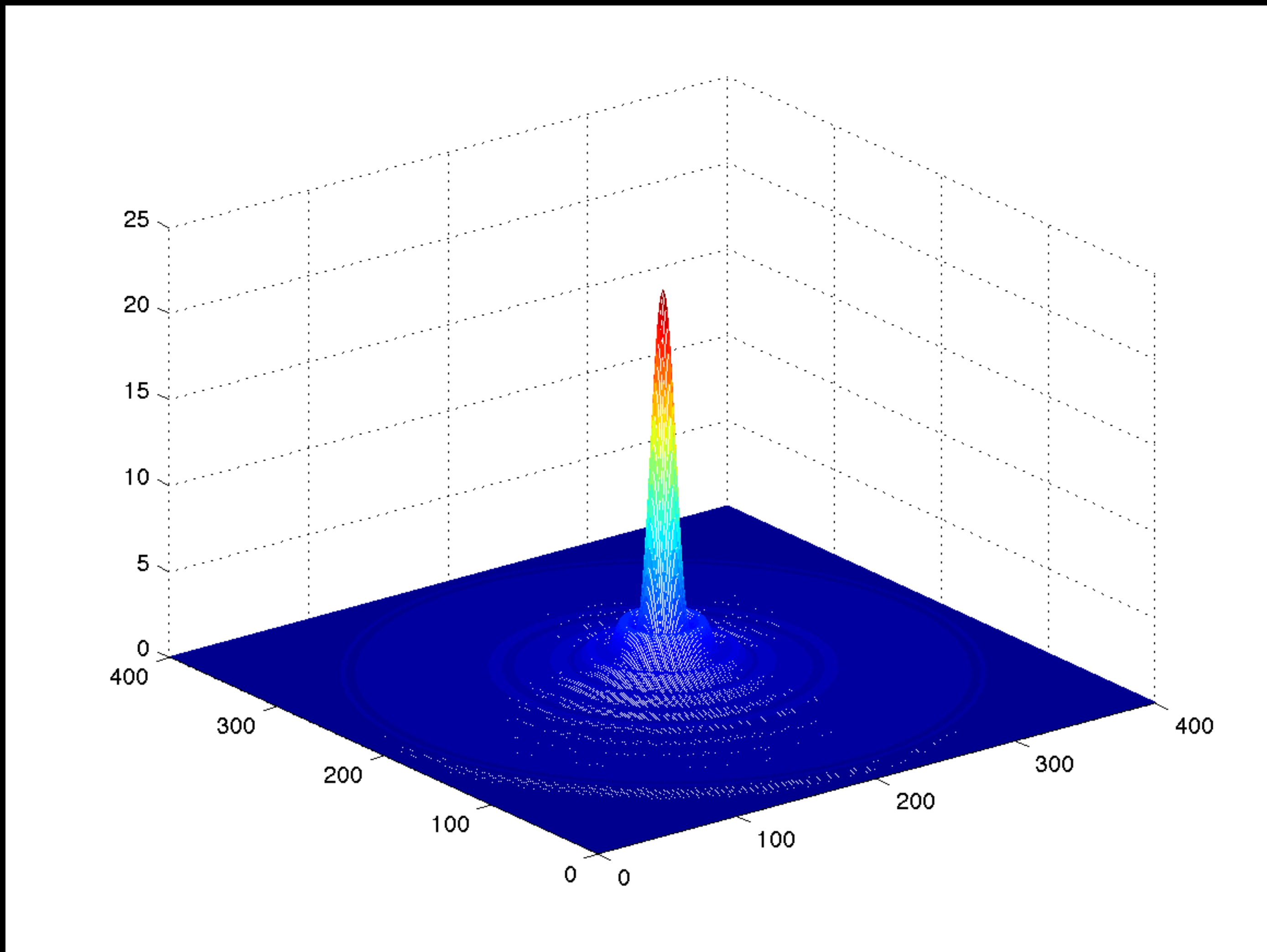
$$J_0(k|\mathbf{x}|), k = 4$$



$$J_0(k|\mathbf{x}|), k = 5$$



$$G(\mathbf{x}) = \int_0^\infty J_0(k|\mathbf{x}|)\,dk$$

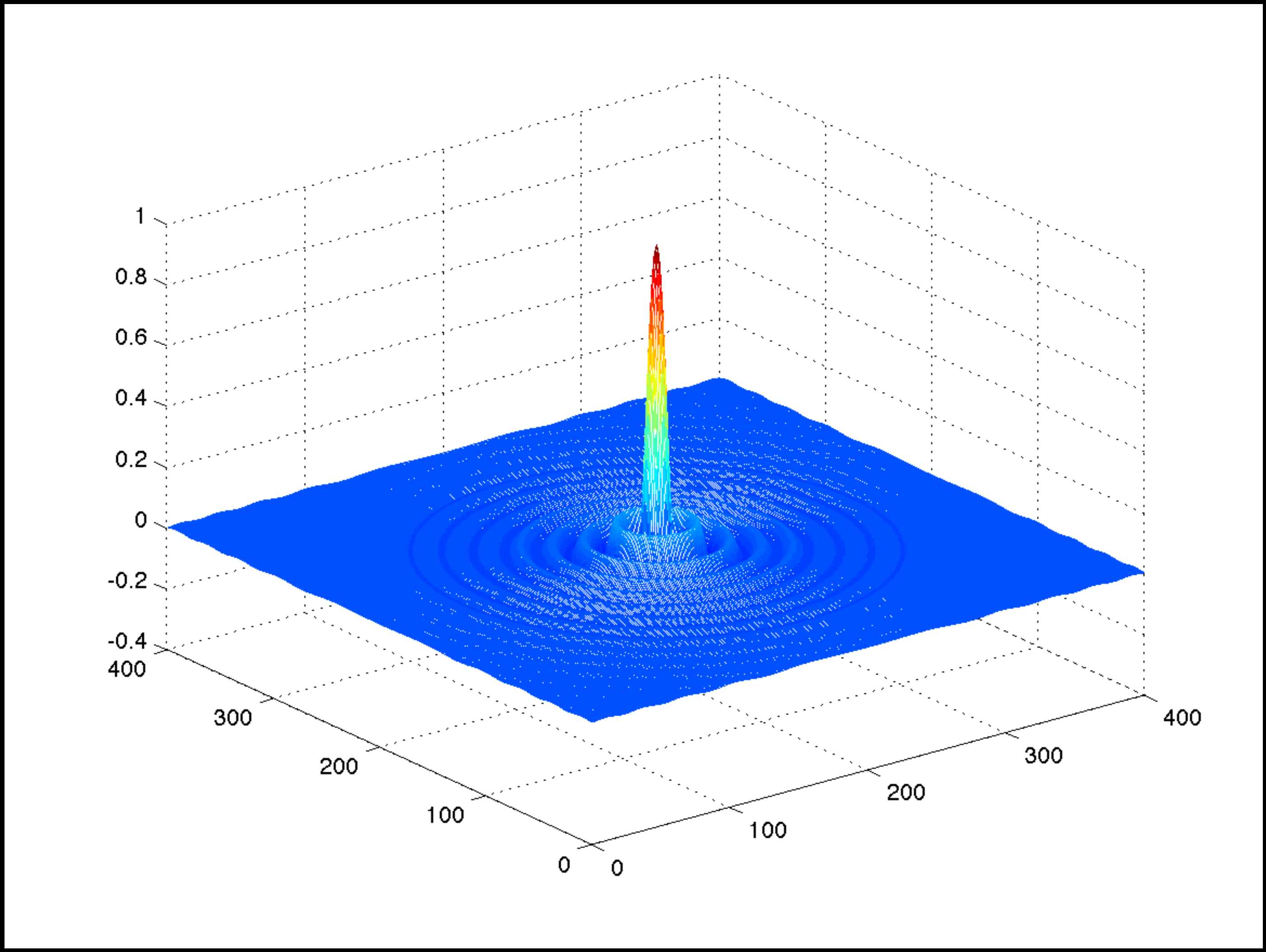


$$G(\mathbf{x}) = \int_0^\infty J_0(k|\mathbf{x}|)\,dk$$

$$\Downarrow$$

$$G(\mathbf{x}) = \int_0^\infty k^2 J_0(k|\mathbf{x}|)\,dk$$

$$G(\mathbf{x}) = \int_0^\infty k^2 J_0(k|\mathbf{x}|)\,dk$$

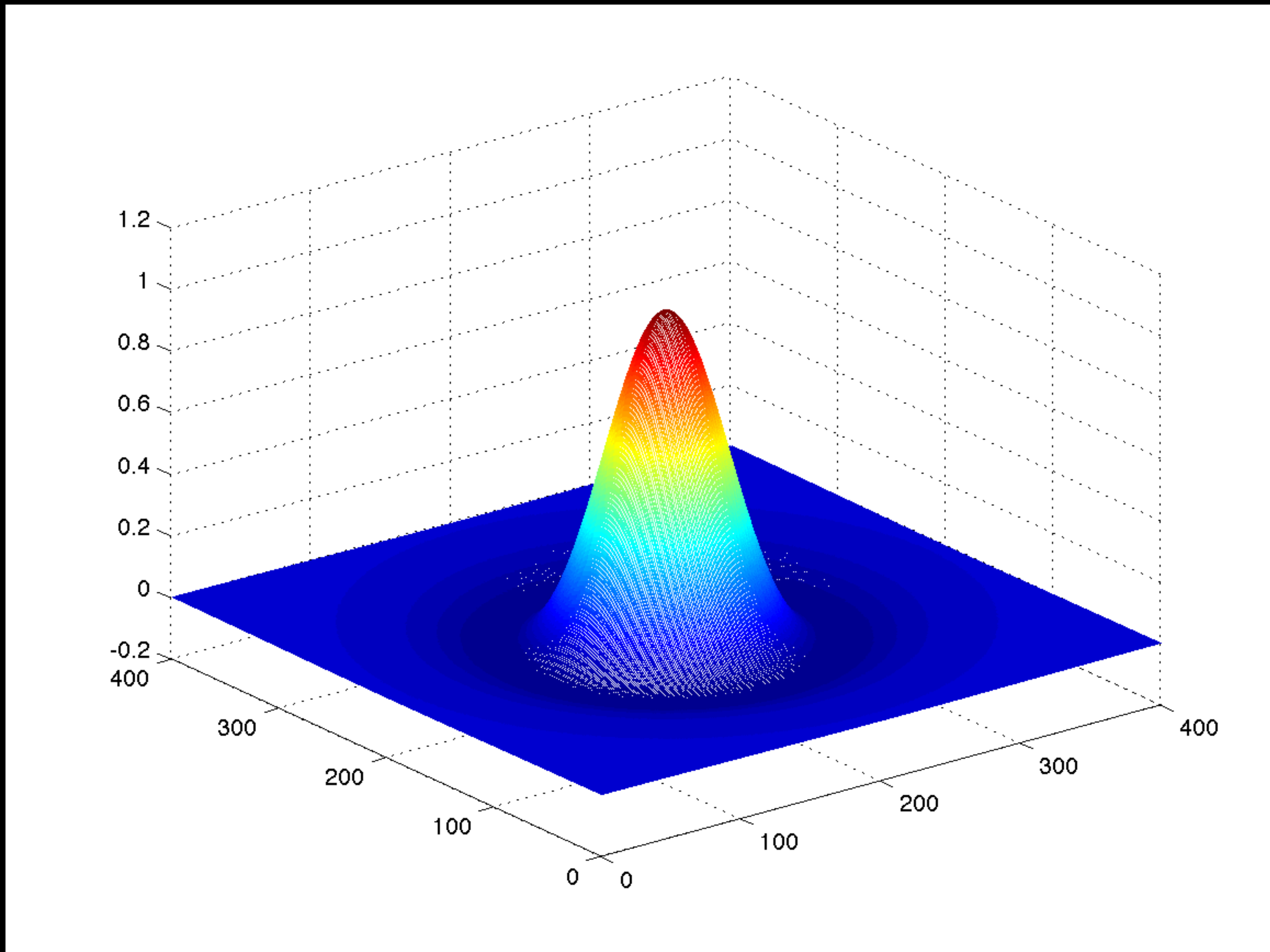


$$G(\mathbf{x}) = \int_0^\infty k^2 J_0(k|\mathbf{x}|)\,dk$$

$$\Downarrow$$

$$G(\mathbf{x}) = \int_0^\infty k^2 e^{-k^2\sigma^2} J_0(k|\mathbf{x}|)\,dk$$

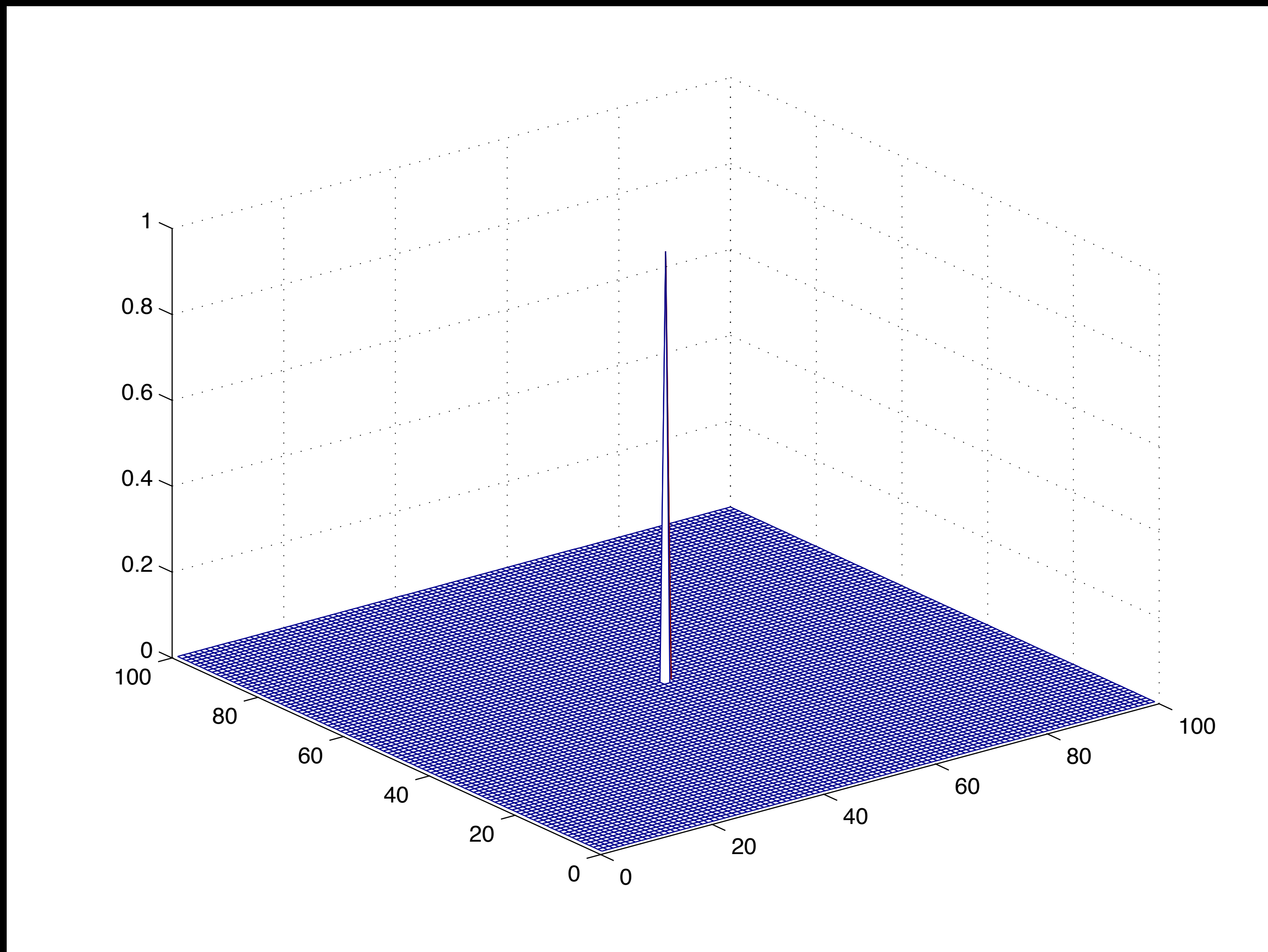
$$G(\mathbf{x}) = \int_0^\infty k^2 e^{-k^2\sigma^2} J_0(k|\mathbf{x}|)\,dk$$

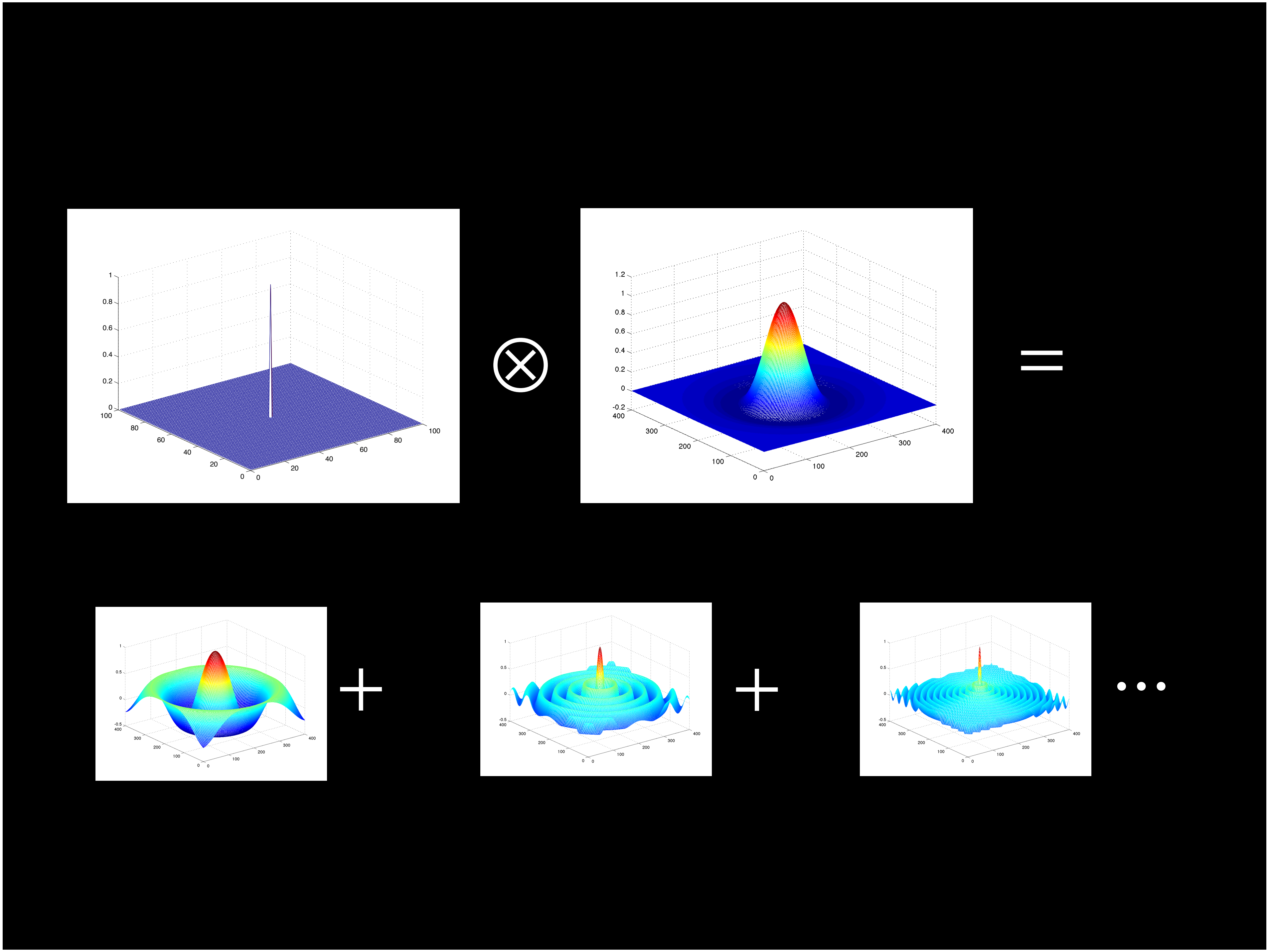


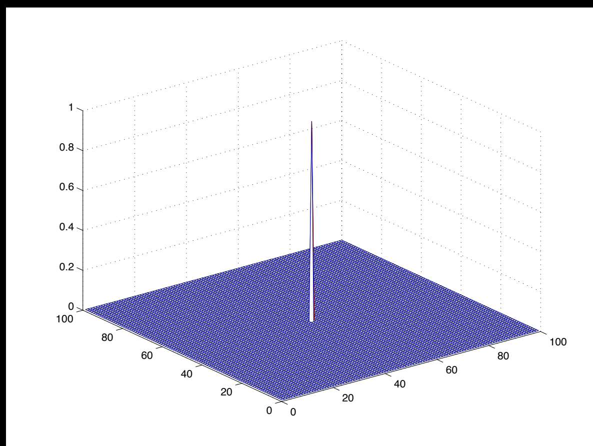
The iWave Equation [Tessendorf 2004]

$$\frac{\partial^2 h}{\partial t^2} = c\sqrt{-\nabla^2}\, h$$

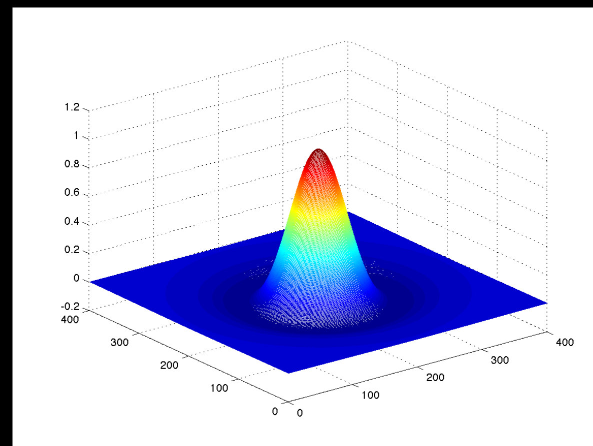
- Dispersion
- ~~Phase shifting~~


1
0.8
0.6
0.4
0.2
0
100
80
60
40
20
0
0
20
40
60
80
100

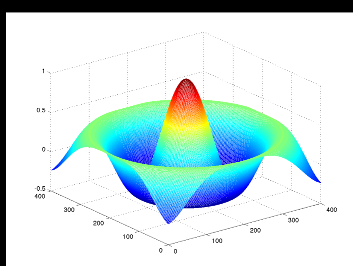
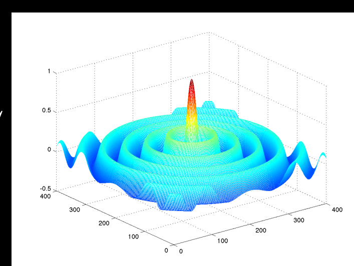
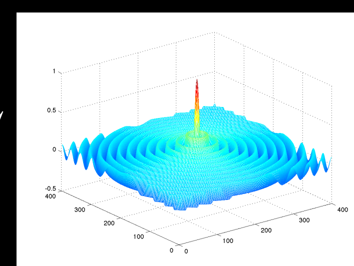


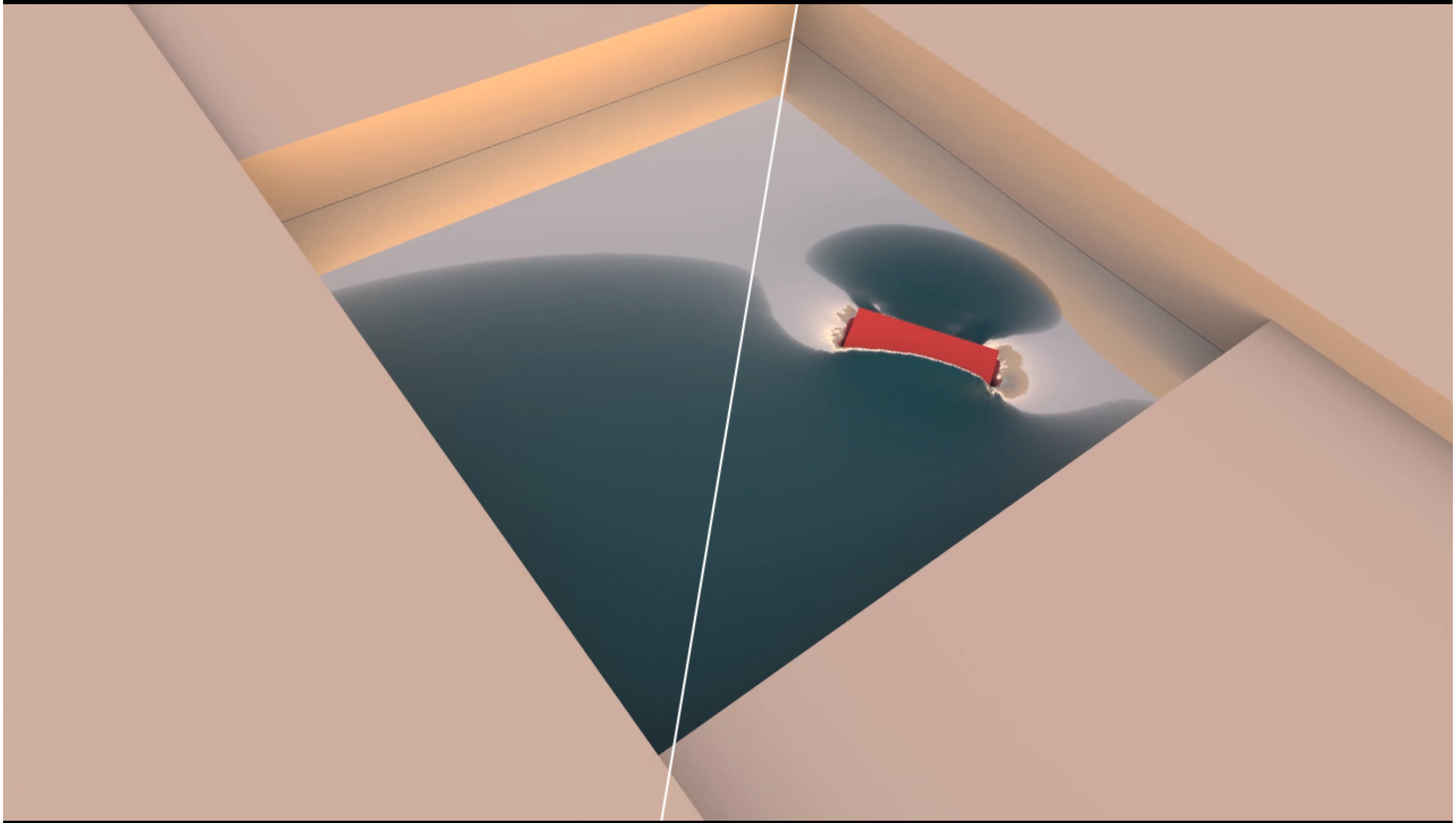


$\otimes$



$=$

1^2  $+\ 2^2$  $+\ 3^2$  $\cdots$



8x up-res

Original

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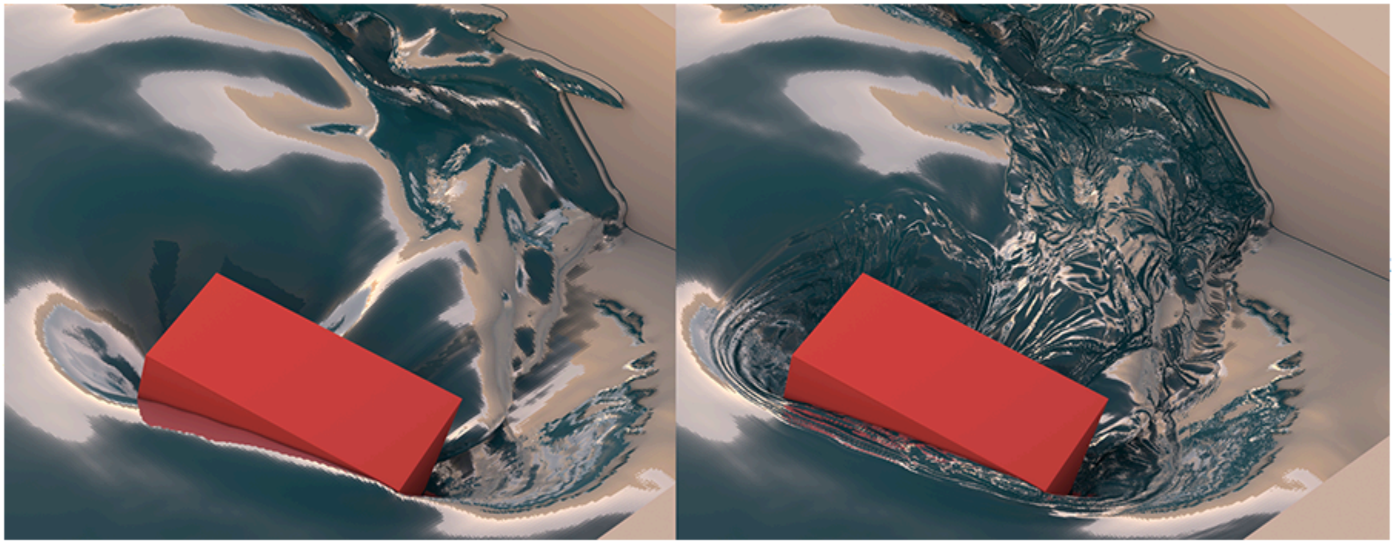
Closest Point Turbulence Source Code

Back to the paper



Download the code: CPT_source.tar.gz
This code is release under the GNU Public License. If you need a less restrictive license, contact us.

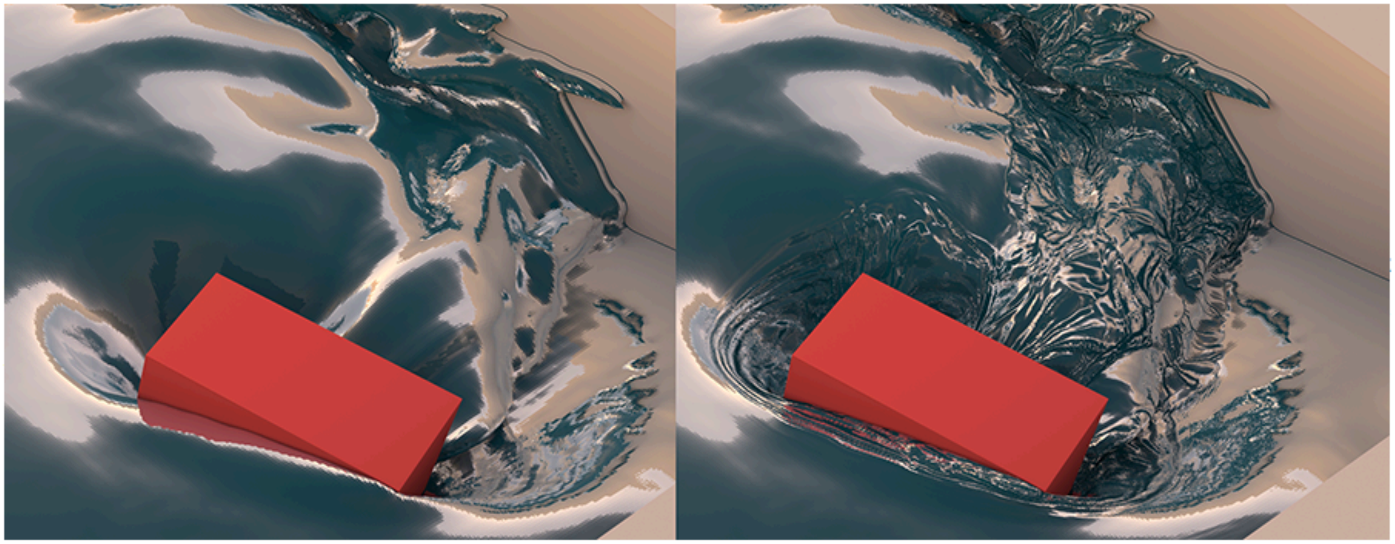
Building the Code

This code is a reference implementation of our paper Closest Point Turbulence for Liquid Surfaces. We have successfully built this code in Mac OSX (10.6, Snow Leopard) and Linux (Ubuntu 12.02). The default `Makefile` is for OSX, and the Linux Makefile is `Makefile.ubuntu`. We have made an effort to remove dependencies on external libraries, so aside from some commonly available libraries (zlib, libpng), you should not need to download and install anything special in order to successfully compile.

http://www.mat.ucsb.edu/~kim/CPT/source.html

Closest Point Turbulence Source Code

Back to the paper



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http://www.mat.ucsb.edu/~kim/CPT/data.zip

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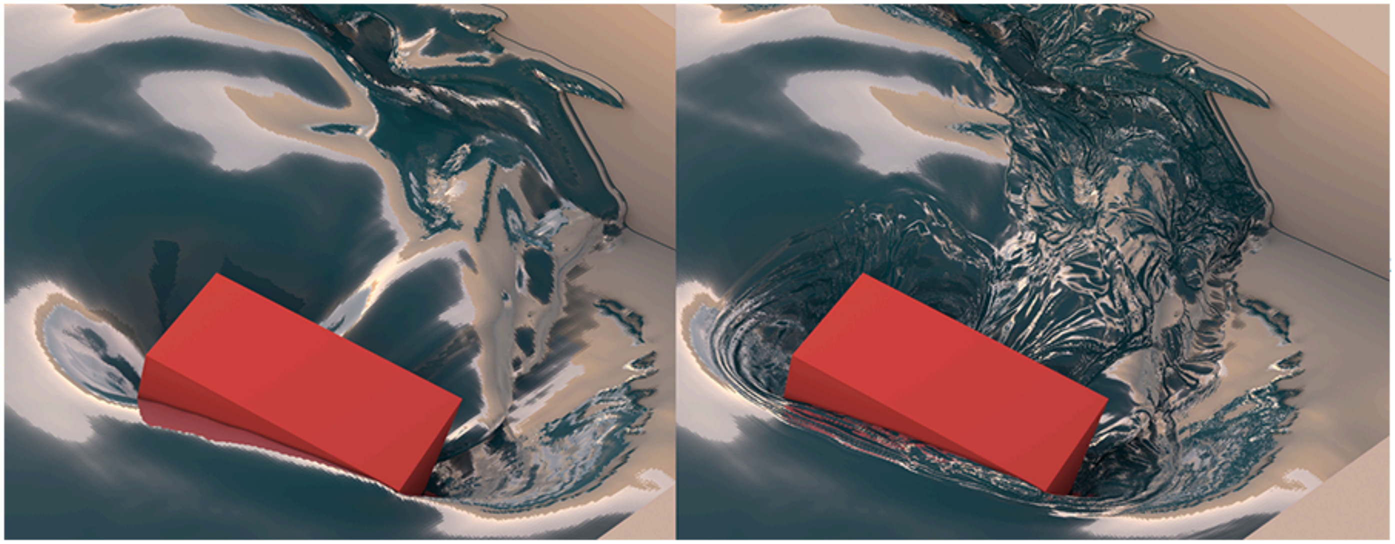

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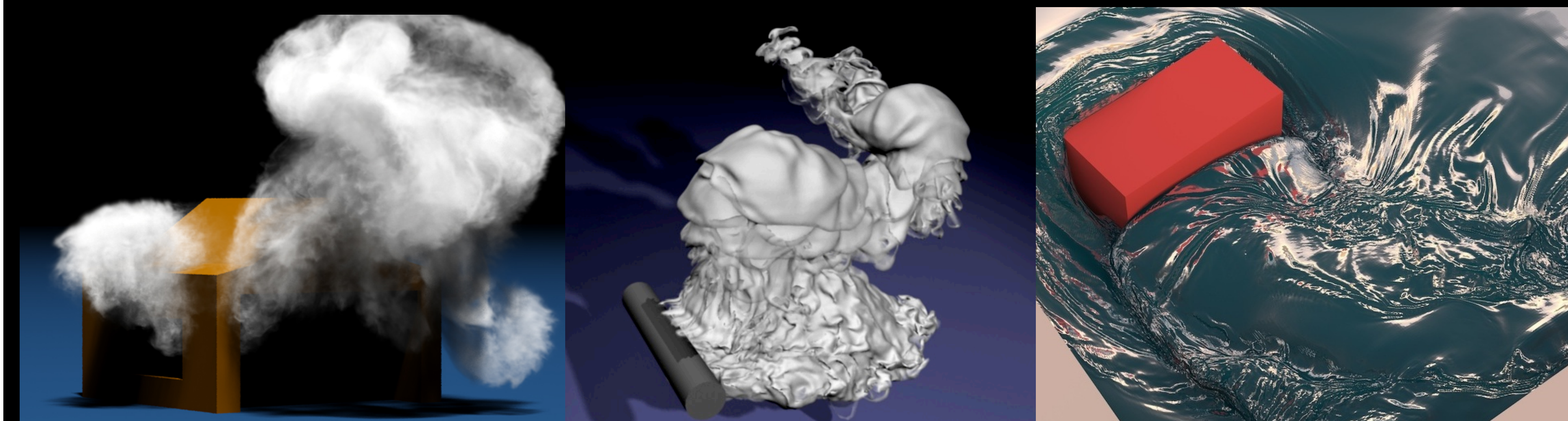
Questions?



Final Thoughts

Final Thoughts - Nils

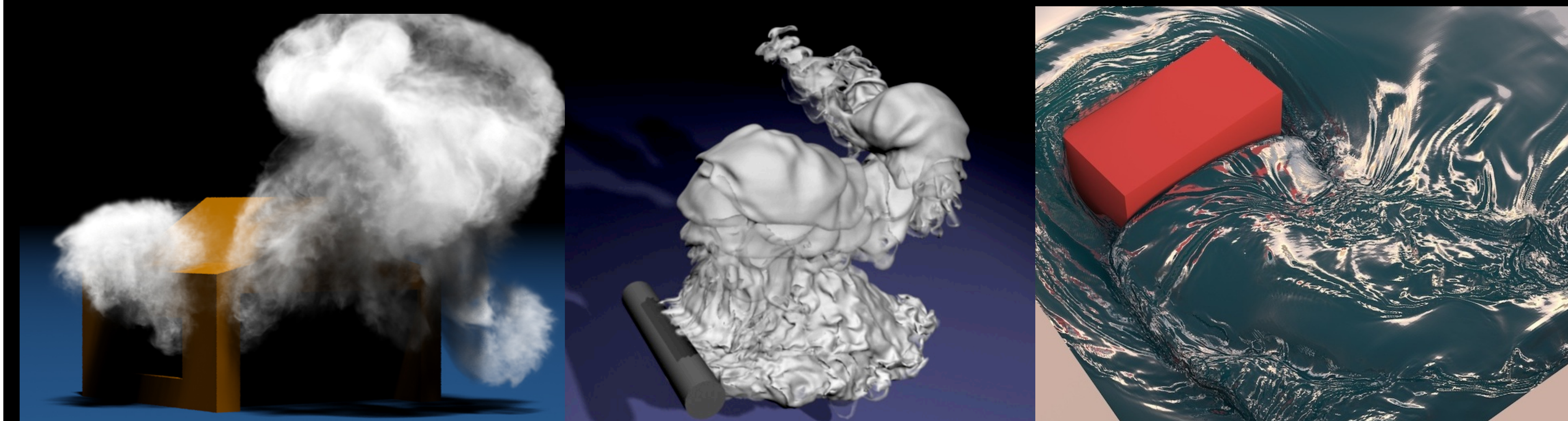
- Artistic control
- Unified turbulence models
- Realism
- Correctness

Final Thoughts - Tobias

- Single phase turbulence prediction: mostly solved
- We need better synthesis
- Detail enhancement: hybrid models

Final Thoughts - Theodore

- Refine the density directly?
  - *Scalar* turbulence, KOC spectrum
- Subdivision based on soliton solutions?
- Capture dispersion and phase shifting?
- An up-res that changes the liquid topology?



THANK YOU

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